

# Fuzzy Classification System for Glass Data Classification

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**Abstract-** *Generating fuzzy rules from training data is the most vital assignment in design of fuzzy classification system. In this paper, we present an approach to deal with the classification problem where fuzzy logic is used. We intend to show that fuzzy logic introduces new elements in the identification process, to manage imprecise information. A method to generate set of definitive fuzzy rules from initial training data is introduced. A triangular membership function is used for generating fuzzy rules from training data as they are simpler and more human understandable with high interpretability.*

## I. INTRODUCTION

The fuzzy classification system is one of the important applications of fuzzy set theory [11]. We proposed a procedure for generating fuzzy rules from input dataset and then to construct a set of definitive rules that are generalizations of initial rules. Fuzzy rules are used for knowledge representation. Two methodologies to get hold of fuzzy rules for fuzzy classification systems. One is given directly by experts; and the other is produced through an automatic learning process. The main purpose of this paper is to obtain an automatic procedure able to get the structure of a fuzzy rule from a given input dataset. Fuzzy rules generated must contain fewer components in the antecedent clause of the rule and identifying simultaneously the largest number of examples in given input data set.

In recent years, some methods as in [4] have been presented to generate fuzzy rules from training data. As in [7], Castro et al, anticipated a method for learning maximal structure rules for dealing with the Iris data classification problem. As in [5] Chang et al, presented a scheme to generate fuzzy rules from numerical data based on the elimination of attribute terms for dealing with the Iris data classification problem. As in [6] Chen et al offered a method for constructing fuzzy decision trees and generating fuzzy classification rules from training dataset.

As in [9] Kasabov, presented method for learning fuzzy rules and approximate reasoning in fuzzy neural networks and hybrid systems. As in [10] Hayushi et al., anticipated a fuzzy neural expert system with automated extraction of fuzzy if-then rules from a trained neural network. As in [8] Nozaki et al, presented a heuristic method for generating fuzzy rules from numerical sets.

In this paper, we present a new method to generate fuzzy rules from a set of training data to deal with the Glass data classification problem. First, we convert the training data into

initial fuzzy rules, and then we merge these initial fuzzy rules in order to reduce the number of initial fuzzy rules. Eventually we

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will get set of definitive fuzzy rule from initial fuzzy rules using different operations like merging, amplification etc. The rest of this paper is organized as follows.

In Section II, we briefly describe a fuzzy rule-based classification [1]. In Section III, present a new method for generating fuzzy rules from training data. In Section IV an example to illustrate the proposed method. The conclusion is discussed in Section V.

## II. FUZZY RULE-BASED CLASSIFICATION

Fuzzy rules are simply IF-THEN rules, used for knowledge representation with high interpretability [1]. For a pattern classification problem, Fuzzy IF-THEN rules include two clauses viz. antecedent and consequent. Antecedent clause includes conditions for the occurrence of the event; while consequent contain consequence of antecedent clause.

For generating fuzzy rules we need to draw membership function for corresponding input data. The length of membership function is obtained using the difference between maximum and minimum value of the attribute. Membership function rescaled each input attribute to unit interval [0, 1] by using linear transformation that preserves the distribution of training patterns. Then, partitioning the pattern into fuzzy subspaces took place where each subspace is identified by a fuzzy rule. By assigning linguistic values of each input attribute we can do partitioning. Generally, triangular membership functions are used for this purpose, as they are simpler and more human understandable with high interpretability [1].

Fig. 1 shows membership functions for four different values of  $K$ , where  $L_3$ ,  $L_4$  and  $L_5$  are the linguistic labels, which interprets linguistic values *small*, *medium*, and *large*, respectively.

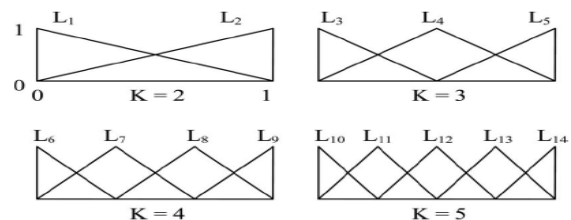


Fig. 1. Different partitioning of each input attributes [1].

Now, consider there are four fuzzy rules:

- If  $x_1$  is *small* and  $x_2$  is *small* then Class 1 ..... (I)
- If  $x_1$  is *small* and  $x_2$  is *large* then Class 2 ..... (II)
- If  $x_1$  is *large* and  $x_2$  is *small* then Class 3 ..... (III)
- If  $x_1$  is *large* and  $x_2$  is *large* then Class 4 ..... (IV)

When fuzzy rule-based systems are used for two-dimensional problems, generated fuzzy rule [3] from above equations is shown in figure2. Where triangular membership function defines small and large linguistic values. It is an example of a fuzzy rule table for a two-dimensional pattern classification problem.

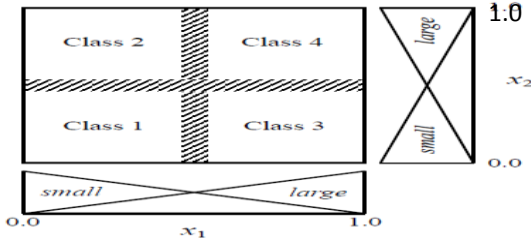


Fig 2. Four fuzzy rules in two-dimensional pattern  $[0, 1] \times [0, 1]$ . Fuzzy IF-THEN rules for a pattern classification problem for  $n$  attributes can be written as follows [1]:

$$\text{Rule } R_j: \text{ IF } x_1 \text{ is } A_{j1} \text{ and } \dots \text{ and } x_n \text{ is } A_{jn} \\ \text{ THEN class } C_j, \quad \text{for } j = 1, N \quad (1)$$

Where  $X = [x_1, x_2, \dots, x_n]$

$\equiv n$ -dimensional pattern vector,

$A_{ji} \equiv$  Antecedent linguistic value of  $R_j$ ,

$C_j \equiv$  Consequent class,

$N \equiv$  Number of fuzzy rules.

Generally, for an  $M$ -class problem with  $m$  labeled patterns  $X_p = [x_{p1}, x_{p2}, \dots, x_{pn}]$ , where  $p = 1, \dots, m$ , our task is to design the classifier to generate  $N$  fuzzy rules as in (1). Using training pattern in the corresponding fuzzy subspace consequent class  $C_j$  of the fuzzy rule  $R_j$  in (1) can be determined. The compatibility grade of the training pattern  $X_p$  is defined with the antecedent part of the rule, by the usage of the product operator as

$$\mu_j(X_p) = \prod_{i=1}^n \mu_{ji}(x_{pi}) \quad (2)$$

Where  $\mu_{ji}(\cdot)$  is the membership function of the antecedent fuzzy set  $A_{ji}$ . Using heuristic method which is based on confidence we select the consequent class of a rule [4]. This confidence is given as

$$\text{conf}(A_j \Rightarrow \text{Class } T) = \frac{\sum_{X_p \in \text{class } T} \mu_j(X_p)}{\sum_{p=1}^m \mu_j(X_p)} \quad (3)$$

Hence from (3), the consequent class  $C_j$  is determined using the maximum confidence, shown using (4).

$$C_j = \arg \max \{ \text{conf}(A_j \Rightarrow \text{class } T) \mid T = 1, \dots, M \} \quad (4)$$

### III. GENERATING FUZZY RULES FROM TRAINING DATA

Our Here, we present a new algorithm to generate fuzzy rules from training data to deal with the Glass data [10] classification problem, where we deal with data are multiple input and single output data (MISO). Take for granted that there are  $m$  instances (i.e.  $a_1, a_2, \dots, a_m$ ) of data to be trained. Assume that the  $i$ th training data  $a_i$  has  $n$  input values and one output value, shown as follows:

$$a_i = ((x_{i1}, x_{i2}, \dots, x_{in}), y_i)$$

Where  $x_{ij}$  is the  $j$ th input value of input variable  $X_j$  of the  $i$ th training data  $a_i$ , and  $x_{ij}$  is a real number;  $y_i$  is the value of output variable  $Y$  of the  $i$ th training data, where  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ .

Let  $I$  be a set of initial rules. We can convert the initial training data into fuzzy rules one to one and put them into the set of initial rules [8]. It can be done in following manner. Apply linguistic label to each input training data, which further represents a membership function. First, we get an initial training data  $a_i$  which is not converted yet. Then, we get the  $j$ th input value of  $a_i$ , denoted by  $x_{ij}$ , which is not converted. Assume that the domain of input variable  $X_j$  was divided into  $z$  equal parts. If  $z = 7$ , we can define 7 labels, as shown in Figure 3, where each label represents a membership function.

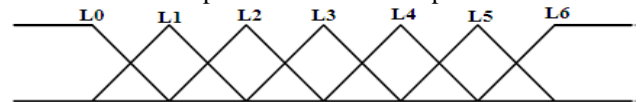


Fig 3. Membership functions of the corresponding labels.

Then, we estimate membership value by mapping  $x_{ij}$  into every membership function as in Figure 3. We compare all of the membership values, take the label which has the maximal membership value, and replace the numerical value  $x_{ij}$  by that label. The same procedure is repeated for getting all labels. Then, we can get a fuzzy rule which is converted from  $x_i$ , and we put this rule into the set of initial fuzzy rules. After this step, we take another initial training data item which is not labeled, repeat the step described above until the all of the training data are converted into fuzzy rules and these fuzzy rules are put into the set of initial rules.

Then, we deal with the fuzzy rules converted from the initial training data set. First, we take a fuzzy rule  $R$  from the set of initial rules. If the set of definitive rules is empty, then we let the fuzzy rule  $R$  be a member of the set of definitive rules; if the set of definitive rules is not empty, then we take one fuzzy rule  $R'$  which has the same output as the fuzzy rule  $R$  and merge it with the fuzzy rule  $R$  [2]. The action of merging is defined as follows.

**Definition 1:** Assume that fuzzy rule  $R_1 = ((L_0, L_1, L_n), y_i)$  and fuzzy rule  $R_2 = ((L'_0, L'_1, L'_n), y_j)$ . Then, fuzzy rule  $R_1$  and  $R_2$  are merged into fuzzy rule  $R_3$ , where  $R_3 = ((L''_0, L''_1, L''_n), y_k)$ , if and only if  $y_i = y_j$ . Then  $L''_0 = L_0 \cup L'_0$ ,  $L''_1 = L_1 \cup L'_1$ , and  $L''_n = L_n \cup L'_n$ , and  $y_k = y_i = y_j$ , where " $\cup$ " is the union operator.

**Example 1:** Assume that fuzzy rules  $P$  and  $Q$  are merged into fuzzy rule  $R$ , where  $P = ((\{L_0, L_5\}, \{L_3\}, \{L_0, L_4, L_2, L_6\}, \{L_1\}), 1)$ ,  $Q = ((\{L_2, L_3, L_6\}, \{L_3\}, \{L_5, L_6\}, \{L_1, L_3, L_4\}), 1)$ , then  $R = ((\{L_0, L_2, L_3, L_5, L_6\}, \{L_3\}, \{L_0, L_2, L_4, L_5, L_6\}, \{L_1, L_3, L_4\}), 1)$ .

**Definition 2:** Assume that fuzzy rules  $P = ((L_0, L_1, L_n), y_i)$  and  $Q = ((L'_0, L'_1, L'_n), y_j)$ . Then, fuzzy rules  $P$  and  $Q$  are in collision, if and only if  $y_i \neq y_j$ ,  $L_0 \cap L'_0 \neq \psi$ ,  $L_1 \cap L'_1 \neq \psi$ , and  $L_n \cap L'_n \neq \psi$ , where  $\psi$  denotes the empty set.

**Definition 3:** The merge of fuzzy rules  $P$  and  $Q$  into fuzzy rule  $R$  is allowed, if and only if fuzzy rule  $C$  does not collide with any of the fuzzy rules which are in the set of initial rules.

If merge of fuzzy rules  $R$  with  $R'$  is allowed, then the fuzzy rule  $R$  is merged with  $R'$  into fuzzy rule  $R''$ , and we use fuzzy rule  $R''$  to replace  $R'$  which is in the set of definitive rules [2]. If the merge of fuzzy rules  $R$  with  $R'$  is not allowed, then take another fuzzy rule which was not taken before and have the same output with fuzzy rule  $R$  and repeat the process described above. If the fuzzy rule  $R$  can't merge with any fuzzy rules in the set of definitive rules, then the fuzzy rule  $R$  becomes a new member of the set of definitive rules. Repeat those processes until all of the fuzzy rules in the set of initial rules.

In this way, we present an algorithm to generate definitive fuzzy rules from a set of training data. [8]

- Step 1: While do examples exist in the set of examples  
Do: To convert the example into an initial rule.
- Step 2: To take an initial rule from the set of initial rules.
- Step 3: To prove if it subsumes in some rule of the set of definitive rules. If it subsumes, go to step 2.
- Step 4: For each variable in the initial rule:  
Step 4.1: For each label not considered  
Step 4.1.1: To prove if it is possible to amplify the rule. If it is not possible, go to Step 4.1  
Step 4.1.2: To amplify the rule.
- Step 5: If there are rules in the set of initial rules, still not considered, go to step 2.  
Or else END.

#### IV. EXAMPLE

In this section, we apply proposed a new method to generate fuzzy rules from training data to deal with the Glass dataset [2] classification problem. The Glass data contain 214 instances. There are six species of the Glass data and nine inputs attributes as shown in Table 2 and Table 3 respectively.

TABLE 2. OUTPUT VALUES OF THE OUTPUT ATTRIBUTES.

Type	Output Classes
1	Building_windows_float_processed
2	Building_windows_non_float_processed
3	Vehicle_windows_float_processed
4	Containers
5	Tableware
6	Headlamps

TABLE 3. THE CHARACTERISTICS OF INPUT ATTRIBUTES OF THE GLASS DATA.

Input Attributes	Min. Value	Max. Value
Refractive Index (RI)	1.5112	1.5339
Sodium (Na)	10.73	17.38
Magnesium (Mg)	0	4.49
Aluminum (Al)	0.29	3.5
Silicon (Si)	69.81	75.41
Potassium (K)	0	6.21
Calcium (Ca)	5.43	16.19
Barium (Br)	0	3.15
Iron (Fe)	0	0.51

In order to clearly illustrate the proposed fuzzy rules generation algorithm [8], We assume that the number of labels for each input attribute is 7, i.e., L0, L1, L2, L3, L4, L5, L6. Here maximum and minimum values for refractive index attribute are 1.5339 and 1.5112 respectively. Hence we plot one membership function shown in fig. 4.

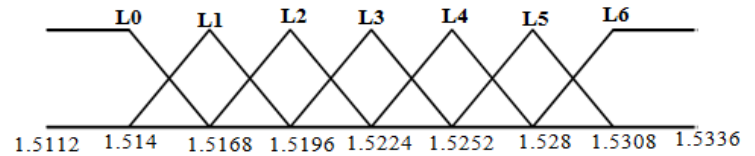


Fig 4. Membership functions of the input attribute.

Similarly we can plot membership function for all remain attributes. Here; we only chose 12 instances from the Glass data for illustration, where 2 instances of each species are chosen. The chosen instances for this example are shown in Table 4. Then, we consider fuzzy rules sequentially from the set of initial rules as in Table 5 and perform the merging process.

Finally, we can get a set of definitive fuzzy rules from the set of initial rules. There are 6 fuzzy rules in the set of definitive rules, shown as follows:

- R0: (({L0}, {L1, L2}, {L2}, {L5, L6}, {L1}, {L1, L3}, {L0}, {L1}, {L0}), 1),
- R1: (({L0}, {L0}, {L0}, {L0}, {L0}, {L0}, {L0}, {L0}, {L0}), 2),
- R2: ({L1}, {L0, L1}, {L2, L3}, {L5}, {L1, L2}, {L2}, {L0}, {L0, L1}, {L0}), 3),
- R3: (({L4}, {L2}, {L2}, {L5}, {L1}, {L3}, {L0}, {L1}, {L0}), 4),
- R4: (({L4}, {L0, L1}, {L1, L2}, {L2, L3}, {L2, L6}, {L0, L3}, {L0, L1}, {L0, L2}, {L0, L4}), 5),
- R5: (({L5}, {L2}, {L1}, {L0, L2}, {L2, L3}, {L2, L3}, {L0}, {L2, L3}, {L0}), 6),

#### V. CONCLUSION

In this paper, we have presented a new method to generate fuzzy rules from training data to deal with the Glass data classification problem. Set of definitive rules is generated from initial fuzzy rules. Fuzzy rules used for knowledge representation. Using Fuzzy IF-THEN rules high interpretability for pattern classification can be achieved. Using triangular membership function accurate fuzzy rules can be obtained.

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TABLE 4. INITIAL TRAINING DATA.

Refractive	Sodium	Magnesium	Aluminum	Silicon	Potassium	Calcium	Barium	Iron	Output/
1.521	13.64	4.49	1.1	71.78	0.06	8.75	0	0	1
1.5176	13.89	3.6	1.36	72.73	0.48	7.83	0	0	1
1.5178	14.84	0.32	0.27	69.83	0.01	5.45	0	0	2
1.522	14.81	0.45	0.3	69.83	0	5.4	0	0	2
1.5157	14.86	3.67	1.74	71.87	0.16	7.36	0	0.12	3
1.5185	13.64	3.87	1.27	71.96	0.54	8.32	0	0.32	3
1.5177	13.65	3.66	1.11	72.77	0.11	8.6	0	0	4
1.5161	13.33	3.53	1.34	72.67	0.56	8.33	0	0	4
1.5151	14.01	2.68	3.5	69.89	1.68	5.87	2.20	0	5
1.5192	12.73	1.85	1.86	72.69	0.6	10.09	0	0	5
1.5206	12.85	1.61	2.17	72.18	0.76	9.7	0.24	0.51	6
1.5212	12.97	0.33	1.51	73.39	0.13	11.27	0	0.28	6

TABLE 5. FUZZY RULES IN THE SET OF INITIAL RULES.

Initial Fuzzy Rules	Initial Fuzzy Rules
(({L0}, {L2}, {L2}, {L6}, {L1}, {L1}, {L0}, {L1}, {L0}),1)	(({L4}, {L1}, {L2}, {L5}, {L1}, {L3}, {L0}, {L1}, {L0}),4)
(({L0}, {L1}, {L2}, {L5}, {L1}, {L3}, {L0}, {L1}, {L0}),1)	(({L4}, {L1}, {L2}, {L5}, {L1}, {L3}, {L0}, {L1}, {L0}),4)
(({L0}, {L0}, {L0}, {L0}, {L0}, {L0}, {L0}, {L0}, {L0}),2)	(({L4}, {L0}, {L2}, {L3}, {L6}, {L0}, {L1}, {L0}, {L4}),5)
(({L0}, {L0}, {L0}, {L0}, {L0}, {L0}, {L0}, {L0}, {L0}),2)	(({L4}, {L1}, {L1}, {L2}, {L2}, {L3}, {L0}, {L2}, {L0}),5)
(({L1}, {L0}, {L3}, {L5}, {L2}, {L2}, {L0}, {L0}, {L0}),3)	(({L5}, {L2}, {L1}, {L2}, {L3}, {L2}, {L0}, {L2}, {L0}),6)
(({L1}, {L1}, {L2}, {L5}, {L1}, {L2}, {L0}, {L1}, {L0}),3)	(({L5}, {L2}, {L1}, {L0}, {L2}, {L3}, {L0}, {L3}, {L0}),6)