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Optimum Replacement Period for Motor Vehicle considering Macroeconomic Parameters

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Abstract - ***The present work is a contribution of mathematical and operations research models that help in arriving at the best replacement strategies to determine an age of motor vehicle at which the replacement is most economical. Parameters like depreciation, money value, forecasted inflation, nominal interest rate and real interest rate are taken into consideration in model development and for various situations, the replacement period to motor vehicle is suggested***

Keywords - **Replacement, Inflation, Forecasting, Interest rates, and Money value**

I. Introduction

The replacement problems are concerned with the situation that arises when the efficiency of item decreases, failure or breakdown occurs. The decrease of efficiency or breakdown may be either gradual or sudden. The objective of replacement is to decide best policy to determine an age at which the replacement is most economical instead of continuing at increased maintenance costs. The problem of replacement is encountered in the case of both men and machines. It is possible to estimate the chances of failure of various ages. The fundamental objective of replacement is to direct the organization for maximizing its profit (or minimizing the cost). Formulation of problem is done for a situation "Replacement model considering forecasted inflation" and a case study is conducted considering the motor vehicle maintenance data

II. Forecasting Technique

Forecasts are estimates of the occurrence, timing or magnitude of future events and forecasting is a technique, which is used for estimating. Forecasting give operations manager a rational basis for planning and scheduling activities, even though actual demand is quite uncertain. Everyday the managers have to take decisions in the face of uncertainty, without knowing what would happen in future. The manager strives to reduce this uncertainty and make better estimates of what is likely to happen in future. This is what forecasting aims to accomplish. Thus, whether it is inventory control, marketing strategy formulation, financial planning, production planning or any other such area of operations, managers have to employ the tool of forecasting

III. Nominal Interest Rate, Real Interest Rate and Inflation Forecasting Technique

Nominal interest rate refers to the interest rate on an investment or loan without adjusting for inflation. The nominal interest rate is simply the interest rate stated on the loan or investment agreement. Real interest rate refers to an interest rate that has been adjusted to remove the effects of inflation to reflect the real cost of funds to the borrower, and the real yield to the lender. The real interest rate of an investment is calculated as the amount by which the nominal interest rate is higher than the inflation rate.

Real Interest Rate = Nominal Interest Rate - Inflation (Expected or Actual). In ordinary language inflation means a process of rising prices. According to Irving Fisher "Inflation occurs when the supply of money actively bidding goods and services increases faster than the available supply of goods". Inflation leads to Inflationary spiral. When prices rise, workers demand higher wages. Higher wages leads to higher costs. Higher costs lead to higher prices. Higher prices again lead to higher wages, higher costs and so on. Thus prices, wages, costs chase each other leading to hyperinflation. Recession is just opposite of inflation and is a state of declining prices due to reduction of total expenditure of the community and this occurs when demand is less than supply of goods and services. According to Quantity of money theorists, inflation is caused by excessive issue of money. According to demand and supply theorists, it is caused by total demand exceeding the total supply of goods and services.

Irving Fisher theory of interest rates relates the nominal interest rate "i" to the rate of inflation "ϕ" and the real interest rate "r". The real interest rate "r" is the interest rate after adjustment for inflation. It is the interest rate that lenders have to have to be willing to loan out their funds. The relation Fisher posted between these three rates is :

$$(1 + i) = (1 + r)(1 + \phi) = 1 + r + \phi + r\phi$$

This is equivalent to $i = r + \phi(1 + r)$

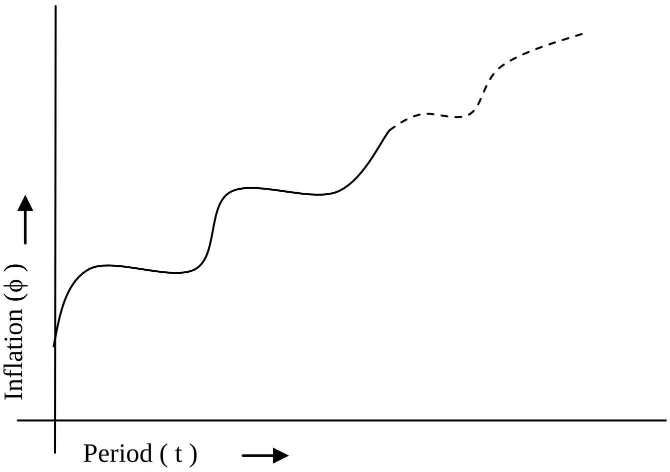
Thus, according to this equation, if 'ϕ' increases by 1% the nominal interest rate increases by more than 1%. This means that if 'r' and 'ϕ' are known then 'i' can be determined. On the other hand, if 'i' and 'ϕ' are known then 'r' can be determined by the relationship is

$1+r = (1+i) / (1+\phi)$ or $r = (i - \phi) / (1+\phi)$

III. Regression Model with Trigonometric Function to Accommodate Cyclical Fluctuation of Prices of Items/Inflation

In the current model it is assumed that inflation which plays vital role in calculation of real interest rate is assumed to be a regression model with trigonometric function to accommodate cyclical fluctuation of prices. Forecasting of inflation is carried out for 20 years based on the regression equation generated.

Variation of Inflation



A model for a seasonal variation might include transcendental functions. The cycle of the model is as below . The model might be used to represent data for the four seasons of the year.

$\phi = b_0 + b_1x + b_2 \sin (2t \pi /4) + b_2 \cos(2t \pi /4)$

Regression equation with trigonometric function is:

$\phi = b_0 + b_1x + b_2 \sin (2t \pi /4) + b_2 \cos(2t \pi /4)$

Here 'ϕ' is inflation, 'x' is a variable and 't' period, b_0,b_1,b_2 are the coefficients. TABLE 3.1 gives the quarterly data of inflation for two years.

TABLE 3.1 Quarterly data of Inflation

Year (t)	0	1/4	1/2	3/4	1	5/4	3/2	7/4	2
ϕ	4.0	4.35	4.2	4.15	4.4	4.75	4.6	4.45	4.8

Following set of equations are used to obtain the values of coefficients of regression model

(1) $\phi = b_0 + b_1x + b_2 \sin (2t \pi /4) + b_2 \cos (2t \pi /4)$

(2) $\sum \phi = nb_0 + b_1 \sum x + b_2 \sum \sin (t \pi /2) + b_2 \sum\cos (2t \pi /4)$

(3) $\sum (\phi x) = b_0 \sum x + b_1 \sum x^2 + b_2 \sum x. \sin (2 \pi t/4) +$
$b_2 \sum x. \cos (2t \pi (2t \pi /4)/4$

(4) $\sum (\phi x^2) = b_0 \sum x^2 + b_1 \sum x^3 + b_2 \sum x^2. \sin (2 \pi t/4) +$
$b_2 \sum x^2. \cos \cos (2t \pi /4)$

When time origin is taken between middle of years the equation reduces to

(5) $\sum \phi = n b_0 + b_2\sum \sin (2 \pi t /4) + b2 \sum \cos (2 \pi t /4)$

(6) $\sum (\phi x) = b_1 \sum x^2$

(7) $\sum (\phi x^2) = b_0 \sum x^2 + b_2 \sum x^2. \sin (2\pi t /4) +$
$b_2 \sum x^2. \cos (2\pi t /4)$

by solving above equations we get the values of regression coefficients

$b_0 = 4.413$, $b_1 =0.0817$, $b_2 = -0.022$

The final regression equation is:

$F_n = 4.413 + 0.082(x) - 0.022\sin(2\pi t/4) - 0.022 \cos(2\pi t/4)$

Accuracy of model is measured by using TABLE-3.0.2 and here total number of periods, N = 9

1. Mean Error = ME = ($\sum$e) / N = 0.0520
2. Mean Absolute Deviation = MAD = ($\sum |e|$) / N = 0.1262
3. Mean Square Error = MSE = ($\sum e^2$) / N = 0.0325
4. Standard deviation of errors = SDE = $\sqrt{[(\sum e^2) / (N-1)]}$ = 0.1914
5. Percentage error = PE = (e / ϕ) * 100
6. Mean percentage error = MPE = $\sum$ PE / N = 1.6148

Mean absolute percentage error = MAPE = $\sum$ | PE | / N = 2.3302

TABLE 3.2 Measurement of Accuracy

x	ϕ	F	e = (ϕ - F)	| e |	(e / ϕ) * 100	(| e | / ϕ)* 100	e^2
-4	4.00	4.0580	-0.0580	0.0580	-1.45	1.45	$3.364x10^{-3}$
-3	4.35	3.9486	0.4016	0.4016	9.23	9.23	0.1612
-2	4.20	4.2088	-0.0088	0.0088	-0.209	0.209	$7.74x10^{-5}$
-1	4.15	4.2154	0.0650	0.0650	-1.56	1.56	$4.225x10^{-3}$
0	4.40	4.3780	0.0220	0.0220	0.5	0.5	$4.84x10^{-4}$
1	4.75	4.4680	0.2810	0.2810	5.91	5.91	0.07896
2	4.60	4.5600	0.0400	0.0400	0.86	0.86	$1.6x10^{-3}$
3	4.45	4.6519	-0.2019	0.2019	0.045	0.045	0.040
4	4.80	4.7420	0.0580	0.0580	1.208	1.208	$3.36x10^{-3}$

TABLE 3.3 Forecasted Inflation for 20 years

x	t (Period in years)	ϕ	x	t (Period in years)	ϕ
-4	0.00	4.00	40	11.00	7.622
0	1.00	4.40	44	12.00	7.898
4	2.00	4.80	48	13.00	8.218
8	3.00	5.062	52	14.00	8.582
12	4.00	5.338	56	15.00	8.902
16	5.00	5.658	60	16.00	9.178
20	6.00	6.022	64	17.00	9.498
24	7.00	6.342	68	18.00	9.862
28	8.00	6.618	72	19.00	10.182
32	9.00	6.938	76	20.00	10.458
36	10.00	7.302			

IV. Replacement of Vehicle Considering Money value (Using Forecasted Inflation and Real Interest Rate) and Depreciation (Diminishing Balance Method):

In this model depreciation value of motor vehicle is considered for a period of 15 years and the depreciation of vehicle at the end of each year is computed by diminishing balance method. Along with depreciation, real interest rate is also considered in decision making for replacement.

Assumptions & Data:

1. The purchase cost of vehicle is C = Rs 15,05,500/- inclusive of all taxes
2. Value of money is considered
3. The nominal rate of interest is assumed as i=20%
4. Real interest rate $r = (i-\phi)/(1+i)$
5. Present worth factor $\nu = 1/(1+r)$
6. Scrap value = S = Rs 5,00,000/-
7. Estimated life = N = 15 years
8. Discount factor $D_f=1-(S/C)^{1/N}$
   $D_f = 1-(500000/1505000)^{1/15} = 0.07$

Calculation of depreciation by diminishing balance method:
Initial cost = C,
Scrap=S,
Estimated life = N
Discount factor $D_f = 1-(S/C)^{1/N}$

$$D_f = 1-(500000/1505000)^{1/15}=0.07$$

TABLE 4.1- Depreciation for 20 years

Year n	Book value Cn=Cn-1(1-Df)	Depreciation Dn=C-Cn
1	1399650	105350
2	1301674.5	203325.50
3	1210557.285	294442.715
4	1125818.275	379181.725
5	1047010.996	457989.004
6	973720.23	531279.77
7	905559.81	599440.2
8	842170.62	662829.4
9	783218.68	721781.32
10	728393.37	776606.63
11	677405.83	827594.17
12	629987.42	875012.58
13	585888.302	919111.698
14	544876.12	960123.88
15	506734.79	998265.21
16	471263.35	1033736.65
17	438274.92	1066725.08
18	407595.69	1097404.33
19	379063.98	1125936.0
20	352529.5	1152470.49

TABLE 4.2 Calculation of average annual cost with money value (Forecasted inflation and real rate of interest) and depreciation

(1) n	(2) ϕ_n	(3) r_n	(4) ν	(5) ν^{n-1}	(6) $\sum \nu^{n-1}$	(7) R_n
1	4.40	2.88	0.26	1	1.000	176000
2	4.80	2.62	0.28	0.28	1.280	192000
3	5.06	2.46	0.29	0.08	1.360	193200
4	5.34	2.31	0.30	0.03	1.390	194400
5	5.66	2.15	0.32	0.01	1.402	195600
6	6.02	1.99	0.33	0.003	1.404	196800
7	**6.34**	**1.86**	**0.35**	**0.001**	**1.405**	**201100**
8	6.62	1.76	0.36	0.007	1.406	201700

(8) $R_n \nu^{n-1}$	(9) $\sum R_n \nu^{n-1}$	(10) $C_n \nu^n$	(11) TC = ($C+\sum R_n \nu^{n-1} - \sum C_n \nu^n$)	(12) $TC/\sum \nu^{n-1}$
176000	176000	391902	1289098	1289098
53760	229760	104133.9	1630626.1	12739260.5
15456	245216	36316.72	1713899.3	1260220.06
5832	251048	11258.18	1744789.8	1255244.47
1956	253004	4083.342	1753920.6	1252800.47
767.52	253771.5	1752.696	1757018.8	1251527.04
361.98	**254133.5**	**706.336**	**1758427.1**	**1250926.34**
157.33	254209.8	362.133	1759016.3	1251078.46

V. Replacement of Vehicle With out Considering Money Value:

In this model a case study is conducted for a motor vehicle. Yearly maintenance cost of vehicle is taken for 20 years and the following assumptions are made in identifying the replacement period i.e., to make a decision when it is advisable to replace or change the vehicle. Generally the average annual cost decreases as the period goes on to certain level and then increase in average annual cost takes place. The period at which the average annual cost is minimum is the replacement period.

Assumptions & data:

1. The purchase cost of vehicle is Rs 15,05,500/- inclusive of all taxes
2. Value of money is not considered
3. The scrap value is Rs 5,00,000/-

TABLE 5.1 Calculation of average annual cost with out money value

Year (n)	Operating + Maintenance cost (R_n)	Total cost ($\sum R_n$)	Resale value (S)	Loss due to resaling (C-S)	Total cost P(n)=(C+$\sum R_n$ -S)	Average cost per year P(n)/n
1	176000	176000	500000	1005000	1181000	1181000
2	192000	368000	500000	1005000	1373000	686500
3	193200	561200	500000	1005000	1566200	522066
4	194400	755600	500000	1005000	1760600	440150
5	195600	951200	500000	1005000	1956200	391240
6	196800	1148000	500000	1005000	2153000	358833

Year (n)	Operating + Maintenance cost (R_n)	Total cost ($\sum R_n$)	Resale value (S)	Loss due to resaling (C-S)	Total cost P(n)=(C+$\sum R_n$-S)	Average cost per year P(n)/n
7	201100	1349100	500000	1005000	2354100	336300
8	201700	1550800	500000	1005000	2555800	319475
9	203800	1754600	500000	1005000	2759600	306622
10	206200	1960800	500000	1005000	2965800	301080
11	209550	2170350	500000	1005000	3175350	288668
12	214700	2385050	500000	1005000	3390050	282504
13	224100	2609150	500000	1005000	3614150	278011
14	227100	2836250	500000	1005000	3841250	274375
15	232000	3068250	500000	1005000	4073250	271550
16	240000	3308250	500000	1005000	4313250	269578
17	**255000**	**3563250**	**500000**	**1005000**	**4568250**	**268720**
18	275000	3838250	500000	1005000	4843250	269069
19	300000	4138250	500000	1005000	5143250	270697
20	325000	4463250	500000	1005000	5468250	273412

VI. Results and Conclusion:

From the TABLE 4.2, the average annual cost of vehicle is decreasing gradually up to 7th year and from 8th year it is increasing. So it is advisable to replace the motor vehicle at the end of 7th year. From TABLE 5.1 it is observed that the average annual cost of vehicle is decreasing gradually up to 17th year and from 18th year it is increasing. So it is advisable to replace the motor vehicle at the end of 17th year. In general real time decision making depends on various uncontrollable parameters. In the current work of replacement of motor vehicle, only some macro-economic parameters are considered, but there is a scope to develop a robust model considering mileage, technological changes, government policies (taxes etc.,).

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