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Mathematical Modeling of Spiral Heat Exchanger

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Abstract : Compact Heat Exchangers (CHEs) are increasingly being used on small and medium scale industries. Due to their compact size and efficient design, they facilitate more efficient heat transfer. Better heat transfer would imply lesser fuel consumption for the operations of the plant, giving improvement to overall efficiency. This reduction in consumption of fuel is a step towards sustainable development. This report exclusively deals with the study the spiral heat exchanger.The design considerations for spiral heat exchanger is that the flow within the spiral has been assumed as flow through a duct and by using Shah London empirical equation for Nusselt number design parameters are further optimized.This is accompanied by a detailed energy balance to generate a concise mathematical model.

1) Introduction

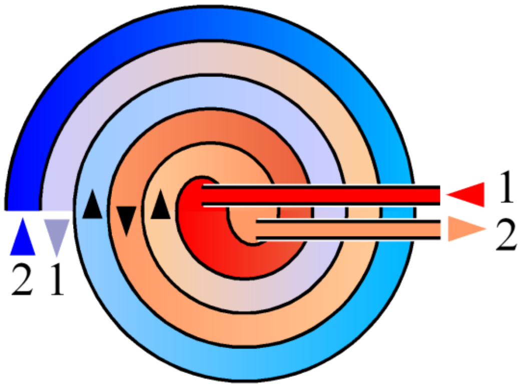
Heat Exchanger is an apparatus that has two streams of fluids at different temperatures that exchange heat in order to satisfy the heating or cooling requirements of the system. They are installed in plants to help improve energy utilization. The energy from the streams at high temperature is transferred to streams at low temperature whenever required. Heat transfer only occurs when the system satisfies both the first and second law of thermodynamics.

A spiral heat exchanger (SHE), may refer to a helical (coiled) tube configuration, more generally, the term refers to a pair of flat surfaces that are coiled to form the two channels in a counter-flow arrangement. Each of the two channels has one long curved path. A pair of fluid ports is connected tangentially to the outer arms of the spiral, and axial ports are common, but optional.

The concept of a spiral heat exchanger is as simple as it is sophisticated. Two or four long metal strips, onto which spacer studs are welded, are wound around a core, thus creating two or four equally spaced single passage channels. The concentric shape of the flow passages and the studs yield turbulence already at low Reynolds numbers. By optimizing the flow pattern heat transfer is enhanced, whilst fouling is reduced.

The SHE is its highly efficient use of space. This attribute is often leveraged and partially reallocated to gain other improvements in performance, according to well-known tradeoffs in heat exchanger design. A compact SHE may be used to have a smaller footprint and thus lower all-around capital costs, or an over-sized SHE may be used to have less pressure drop, less pumping energy, higher thermal efficiency, and lower energy costs.

The most fascinating feature of SHE is that due to its geometry, there is very limited fouling. As the fluid flows in a spiral, deposition on the walls f the heat exchanger is negligible. This means lower down time as compared to the conventional Shell and Tube heat exchangers.



(Fig 2.1) conceptual flow of fluids (hot and cold) in SHE, courtesy Alfa Laval)

While designing any heat exchanger for a plant, the heat flux, temperature drop and the surface area required to transfer the heat is known via the preliminary Heat Exchanger Network calculations. The design parameters aim to achieve these requirements in the most optimal way plausible keeping in mind safety, operability, maintainability, sustainability and profitability. In this paper, I assume that the heat flux and the inlet-outlet temperature of both hot and cold streams are known.

2) Methodology

For the design equation calculation, a differential cross section is considered for the spiral heat exchanger and then used counter current flow **LMTD** method to solve for the design parameters.

$$(LMTD) = \frac{(T_{h,out} - T_{c,in}) - (T_{h,in} - T_{c,out})}{\ln\left(\frac{(T_{h,out} - T_{c,in})}{(T_{h,in} - T_{c,out})}\right)} \qquad (Eq1.1)$$

$T_{h,in}$ = **Hot Stream Inlet Temperature**
$T_{h,out}$ = **Hot Stream Outlet Temperature**
$T_{c,in}$ = **Cold Stream Inlet Temperature**
$T_{c,out}$ = **Cold Stream Outlet Temperature**

$$q = UA\ (LMTD) \qquad (Eq1.2)$$

Here, the heat flux is represented as a function LMTD.

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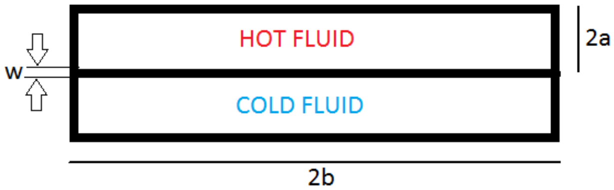
Design: The distance between the sheets in the spiral channels are maintained by using spacer studs that were welded prior to rolling. Once the main spiral pack has been rolled, alternate top and bottom edges are welded and each end closed by a gasketed flat or conical cover bolted to the body. This ensures no mixing of the two fluids occurs.



(Fig 2.2) CROSS SECTION CONSIDERING HOT, COLD ELEMENTS EACH.

Using LMTD (log mean temperature difference) Method[4]

Assuming counter current flow of fluids

Heat balance equation (taking LMTD approach)

$$dq = -\dot{m}_h C_h dT_h = \dot{m}_c C_c dT_c = U(T_h - T_c)dA \quad \text{(Eq2.1)}$$

$$dT_h - dT_c = -dq\left(\frac{1}{\dot{m}_c C_c} + \frac{1}{\dot{m}_h C_h}\right)$$

$$d(T_h - T_c) = -U(T_h - T_c)\left(\frac{1}{\dot{m}_c C_c} + \frac{1}{\dot{m}_h C_h}\right)dA$$

$$\int_1^2 \frac{d(T_h - T_c)}{(T_h - T_c)} = \int_0^A -U\left(\frac{1}{\dot{m}_c C_c} + \frac{1}{\dot{m}_h C_h}\right)dA$$

$$\ln\left(\frac{(T_{h,out} - T_{c,in})}{(T_{h,in} - T_{c,out})}\right) = -UA\left(\frac{1}{\dot{m}_c C_c} + \frac{1}{\dot{m}_h C_h}\right) \quad \text{(Eq2.2)}$$

As,

$$\dot{m}_h C_h = \frac{q}{T_{h,in} - T_{h,out}} \quad \text{(Eq2.3)}$$

$$\dot{m}_c C_c = \frac{q}{T_{c,out} - T_{c,in}} \quad \text{(Eq2.4)}$$

We get,

$$q = UA\frac{(T_{h,out} - T_{c,in}) - (T_{h,in} - T_{c,out})}{\ln\left(\frac{(T_{h,out} - T_{c,in})}{(T_{h,in} - T_{c,out})}\right)}$$

$$q = UA\,(LMTD)$$

Where,

$$(LMTD) = \frac{(T_{h,out} - T_{c,in}) - (T_{h,in} - T_{c,out})}{\ln\left(\frac{(T_{h,out} - T_{c,in})}{(T_{h,in} - T_{c,out})}\right)} \quad \text{(Eq2.5)}$$

Where U can be estimated by considering the thermal resistances offered by the heat exchanger.

Assuming at each location around the spiral, the hot fluid dissipates equal heat to the outer and inner (radial direction).Effectively, each location dissipates energy to a location radially outward from it.

Assuming the flow is through a duct at each location. The resistances (as represented in fig. 2.3) to the heat flux are offered by the thermal resistivity of the separating wall and convective heat flux offered by the flowing fluids. As the heat flux for every compartment flows from the hot fluid to the cold fluid, the overall heat transfer coefficient can be evaluated using the concept of resistances in series. Estimating the over all heat transfer coefficient using this concept.

(Fig 2.3) Block diagram of thermal resistances in series

$$\mathbf{U} = \left(\frac{1}{h_e} + \frac{W}{K_s} + \frac{1}{h_c}\right)^{-1}; \quad \text{(Eq2.6)}$$

The overall heat transfer coefficient

In Eq2.6, **w** is fixed as a design parameter and has fixed values based on material of construction. K_s is fixed as well as it depends on the material of construction which is a design decision. h_e and h_c both are dependent on the flow of the fluid, Nusselt number (a dimensionless number) is used for the estimation of h (convective heat transfer coefficient). The focus here is to estimate overall heat transfer coefficient by relating it to the Nusselt number.

Nusselt number, $Nu = \frac{h.L_{effective}}{K_{fluid}}$

(Definition of Nusselt number)

$$\Rightarrow h = \frac{Nu\, K_{fluid}}{L_{effective}} \quad \text{(Eq2.7)}$$

Now, estimating each term on the R.H.S.

K_{fluid} , is a property of the fluid flowing in each cold and hot side, it has no control on the design parameters.

$L_{effective}$ estimation; as the flow of the fluid at each location can be assumed to be a flow through a rectangular duct

$$L_{effective} = \frac{4A}{P} = \frac{4(2a*2b)}{2(2a+2b)} = D_h$$

As, a << b

$$L_{effective} = 4a \quad \text{(Eq2.8)}$$

Using Shah-London empirical equation [3]

$$Nu_{D_h} = 8.235(1 - 2.0421\epsilon + 3.0853\,\epsilon^2 - 2.4765\,\epsilon^3 + 1.0578\,\epsilon^4 - 0.1861\,\epsilon^5) \quad \text{(Eq2.9)}$$

Where $\epsilon = \frac{a}{b}$

Using (Eq2.8) in (Eq2.7)

$$h = (Nu_{D_h} * K_{fluid})/4a \quad \text{(Eq2.10)}$$

For hot fluid

$$h_h = (Nu_{D_h} * K_{hot,fluid})/4a \quad \text{(Eq2.11)}$$

For cold fluid

$$h_c = (Nu_{D_h} * K_{cold,fluid})/4a \quad \text{(Eq2.12)}$$

Putting (Eq2.11) and (Eq2.12) in (Eq2.6)

$$\mathbf{U} = \left(\frac{4a}{(Nu_{D_h} * K_{hot,fluid})} + \frac{w}{K_s} + \frac{4a}{(Nu_{D_h} * K_{cold,fluid})}\right)^{-1}$$

Putting in (Eq2.5)

$$\mathbf{q} = \mathbf{A}\frac{LMTD}{\frac{4a}{(Nu_{D_h} * K_{hot,fluid})} + \frac{w}{K_s} + \frac{4a}{(Nu_{D_h} * K_{cold,fluid})}} \qquad \text{(Eq2.13)}$$

Nu_{D_h}, is given by the Shah-London empirical equation, and is same for both hot and cold fluid as the size of the duct is same for both.

Here $K_{h,fluid}$, $K_{c,fluid}$, K_s , C_c ,C_h -> are properties and are fixed as a design point of view they are dependent only on the bulk temperature of the fluid.Assuming these parameters do not change much within the working temperature range on the heat exchanger they can be assumed to be constant.
A, is the surface area of the heat exchanger.

Higher the value of A, more will be the log mean temperature difference and hence better heat transfer but larger A means more construction material, and hence a higher capital cost

Optimizing ϵ parameter

$q \alpha Nu_{D_h}$ (Eq2.14)

As, $Nu_{D_h} = f(\epsilon) = 8.235(1 - 2.0421\epsilon + 3.0853\epsilon^2 - 2.4765\epsilon^3 + 1.0578\epsilon^4 - 0.1861\epsilon^5)$ (Eq2.15)

Maximizing by ignoring higher order terms

$$\frac{\partial Nu_{D_h}}{\partial \epsilon} = (-2.0421+6.1706\epsilon - 7.4295\epsilon^2)*8.235$$

For extrema

$$\frac{\partial Nu_{D_h}}{\partial \epsilon} = (-2.0421+6.1706\epsilon - 7.4295\epsilon^2)*8.235 = 0 \qquad \text{(Eq2.16)}$$

Hence

As $\epsilon > 0$

Solving equation (Eq2.15)

$\boldsymbol{\epsilon \approx 0.4153}$

$$\frac{\partial \frac{\partial Nu_{D_h}}{\partial \epsilon}}{\partial \epsilon} = 8.235(6.1706 - 14.859\epsilon)$$

At $\epsilon = 0.4153$

$$\frac{\partial \frac{\partial Nu_{D_h}}{\partial \epsilon}}{\partial \epsilon} = -2.822 * 10^{-3}$$

As

$$\frac{\partial \frac{\partial Nu_{D_h}}{\partial \epsilon}}{\partial \epsilon} = -2.822 * 10^{-3} < 0$$

So, Nu_{D_h} *is maximum at* $\epsilon = 0.4153$

From Eq. 2.15

$\boldsymbol{Nu_{D_h}(0.4153) = 4.172}$ (Eq2.17)

And $\boldsymbol{\epsilon = \frac{a}{b} = 0.4153}$

$\Rightarrow \boldsymbol{a = 0.4153b}$ (Eq2.18)

These design parameters should be used to get the optimum heat transfer.The optimum design equation is:
Substituting Eq2.18 in Eq2.13

$$\boldsymbol{q} = \mathbf{A}\frac{LMTD}{0.3983b\left(\frac{1}{K_{hot,fluid}} + \frac{1}{K_{cold,fluid}}\right) + \frac{w}{K_s}} \qquad \text{(Eq2.19)}$$

Now, estimating the **A** for spiral heat exchangers.

$$A = 2\,(2b)l = 4bl \qquad \text{(Eq2.20)}$$

Estimating length of the spiral:

Using logarithmic spiral equation for estimating the length of the spiral[17]

For a spiral the locus in polar coordinates is:

$$r = \propto e^{\beta\theta} \qquad \text{(Eq2.21)}$$

for the spiral:

$r = 2a$; at $\theta = 2\pi$

$r = 4a$; at $\theta = 4\pi$

Hence, solving for $\propto$ and β we get,

$$r = ae^{\frac{\ln(2)}{2\pi}\theta} \qquad \text{(Eq2.22)}$$

Taking a line integral to estimate the value for **l**

$$l = \int_0^{n\pi} \sqrt{r^2 + (\frac{dr}{d\theta})^2}\, d\theta$$

Where, $\boldsymbol{n}$ is decided by the designer.

On substituting Eq2.22 and solving the integral

$$l = \frac{2a\pi}{\ln(2)}\sqrt{1 + (\frac{\ln(2)}{2\pi})^2}\,(e^{n\frac{\ln(2)}{2}} - 1) \qquad \text{(Eq2.23)}$$

Substituting in Eq2.20;

$$A = 4bl = \frac{8ba\pi}{\ln(2)}\sqrt{1 + (\frac{\ln(2)}{2\pi})^2}\,(e^{n\frac{\ln(2)}{2}} - 1)$$

As, $\boldsymbol{a = 0.4153b}$

$$\Rightarrow A = \frac{3.3224b^2\pi}{\ln(2)}\sqrt{1 + (\frac{\ln(2)}{2\pi})^2}\,(e^{n\frac{\ln(2)}{2}} - 1)$$

Substituting in Eq2.19;

$$\boldsymbol{q} = \frac{\left(\frac{3.3224\,b^2\pi}{\ln(2)}\sqrt{1+(\frac{\ln(2)}{2\pi})^2}\,(e^{n\frac{\ln(2)}{2}}-1)\right)LMTD}{0.3983b\left(\frac{1}{K_{hot,fluid}} + \frac{1}{K_{cold,fluid}}\right) + \frac{w}{K_s}} \qquad \text{(Eq2.24)}$$

In the above equation **q** and **LMTD** are fixed as mentioned before. The only variable is **b** which can be estimated from the above equation. Hence, the entire heat exchanger can be designed.

4) Results and Conclusion

After studying the heat balance equation and applying the LMTD method, the optimum design parameters for a spiral heat exchanger were achieved, the final design equation which is given as

$$\boldsymbol{q} = \frac{\left(\frac{3.3224\ b^2\pi}{\ln(2)}\sqrt{1+\left(\frac{\ln(2)}{2\pi}\right)^2}\left(e^{n\frac{\ln(2)}{2}}-1\right)\right)\boldsymbol{LMTD}}{\boldsymbol{0.3983b}\left(\frac{\boldsymbol{1}}{\boldsymbol{K_{hot,fluid}}}+\frac{\boldsymbol{1}}{\boldsymbol{K_{cold,fluid}}}\right)+\frac{\boldsymbol{w}}{\boldsymbol{K_s}}}$$

This paper dealt with the design of a compact heat exchanger, heat balance was carried out for the spiral heat exchanger taking into account both the conductive and convective modes of heat transfer. The design equation has been represented with physical parameters that decide the dimensions of the entire heat exchanger. The initial heat exchanger network calculations of any plant provides the heat flux and the change in temperatures of the streams, upon feeding those values to the above modeled equations, the design parameters for the heat exchangers can be estimated.

This model helps estimate the width of the spiral 'b'. While developing the equation using the Shah London's equation for Nusselt number which is dependent on 'ϵ' has been optimized to produce a design that would provide the best heat transfer while using minimum material to construct the SHE. By this approach, a more efficient design can be approached. Reducing the material would have direct impact on the capital expense of the heat exchanger.

NOMENCLATURE

$\dot{m}_h$->Mass flow rate of hot fluid (kg/h)
C_h->Specific heat capacity of hot fluid (kJ/Kg.K)
T_h->Temperature of hot fluid (K)
$\dot{m}_c$->Mass flow rate of cold fluid (kg/h)
C_c->Specific heat capacity of cold fluid (kJ/Kg.K)

T_c->Temperature of cold fluid (K)
U->Overall heat transfer coefficient (W/m^{2}k)
A->Total internal surface area of the heat exchanger (m^2)
h_h->Convective heat transfer coefficient for hot fluid (W/m^{2}k)
h_c->Convective heat transfer coefficient for cold fluid (W/m^{2}k)
w->Width of the separating wall (m^2)
K_s->Thermal conductivity of wall (W/mK)
K_{fluid}->Thermal conductivity of the fluid (W/mK)
Nu_{D_h}->Nusselt number for hydraulic diameter D_h
$K_{h,fluid}$->Thermal conductivity of hot fluid. (W/mK)
$K_{c,fluid}$->Thermal conductivity of cold fluid(W/mK).
D_h-> Hydraulic Diameter (m)
l -> length of spiral (m)
n -> Number turns of the spiral

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