

Analytical Models Evaluation for Predicting Vertical Velocity Flows through Submerged Vegetation

Mabrouka Morri, Amel Soualmia, Philippe Belleudy

¹Laboratory of Water Sciences and Technology, National Institute of Agronomy of Tunisia, University of Carthage, Tunis, Tunisia

²Laboratory of Transfers study in Hydrology and Environment, University Joseph Fourier, University of Grenoble, France

Corresponding Email: amel.inat@hotmail.fr

Abstract— this study explores the capacity of different analytic models to predict the vertical velocity profile in vegetated flume with submerged vegetation and select the suitable model. The verification is determined due to a comparison between the measured data in vegetated flume and the simulated velocity profiles using the Scilab software.

Keywords— Velocity Profile, Analytical Models, Vegetation, Prediction.

I. Introduction

Vegetation within rivers and flood plains occurs under different forms; it can be flexible or rigid and submerged or emergent in flows. It plays an important role in the flow structure of many streams and rivers and may increase flood risks. Thus, an understanding of vegetated flows is necessary to control floods and the ecosystem of the stream (Wu and He, 2009; Liu and Shen, 2008; Evangelos, 2012). The effects of vegetation on flow have been studied for many years by laboratory experiments and numerical models with rigid cylinder, flexible vegetation prototypes and natural vegetation on open-channel flow (Shimizu and Tsujimoto, 1994; Lopez and Garcia, 2001; Tsujimoto et al, 1993; Kubrak et al., 2008; Jarvela, 2005; Klopstra et al., 1997; Stone and Shen, 2002; Baptist et al., 2007; Huthoff et al., 2007; Yang et Choi, 2010). Various approaches have been proposed are based on two-layer approach to describe flow through submerged vegetation. In this approach, the flow domain was divided into two layers (Figure 1); one through the vegetation called “vegetation layer” and the other above it called the “Surface layer”. The flow in each of the two layers was described separately. The logarithmic flow velocity profile is adopted for solving the velocity above the vegetation, and the momentum equation within the vegetated layer. The continuity of the velocity and the shear stress between the two layers is ensured by boundary conditions at the interface (Klopstra et al., 1997; Huai et al., 2009; Jarvela, 2005). For non-vegetated flow, the vertical velocity distribution is related directly to the bed shear stress. While, for vegetated flow, it is related to the vegetation drag because the vegetation roughness is much larger than the river bed roughness. Herein, this study compares different analytic models based on two-layer approach to analyze the performance of these models, for predicting the vertical flow velocity profiles through submerged vegetation and to select the suitable model to be applied in a real case of river.

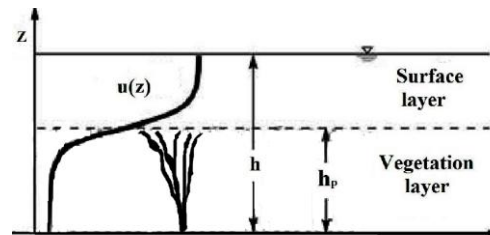


Figure 1. Velocity profile within and above vegetation

II. Theoretical Background

Model 1 klopstra et al.(1997)

Klopstra et al. (1997), proposed an analytical expression for the velocity distribution based on the momentum equation for the vegetation layer assuming uniform steady flow. The vertical velocity distribution in the vegetated layer is given by the following equation:

$$u(z) = \sqrt{C_1 \exp(-\sqrt{2Az}) + C_2 \exp(\sqrt{2Az}) + u_{s0}^2} \quad (1)$$

$$u_{s0} = \sqrt{\frac{2gi}{C_D mD}} \quad (2)$$

$$A = \frac{mD C_D}{2\alpha} \quad (3)$$

Where z is the vertical coordinate, g is the acceleration gravity, i is the energy gradient, m is the density of vegetation, D is the diameter of plant stems, C_D is the drag coefficient, $u(z)$ is the flow velocity at the level z , α is a closure parameter derived from experimental data and u_{s0} is the characteristic constant flow velocity in non-submerged vegetation.

The constants C_1 and C_2 follow from boundary conditions; at the bed ($z = 0$), the bottom shear stress is neglected and the flow velocity is assumed to be equal to u_{s0} , while, at the top of the vegetation layer, the shear stress is determined by the following expression:

$$\tau(h_p) = \rho g(h - h_p)i \quad (4)$$

With τ is the shear stress, ρ is the density of water.

In the surface layer, the velocity follows a logarithmic profile. The connection between the boundary conditions at the

interface ensures the continuity of the velocity and the shear stress between the two layers, and allows the determination of the logarithmic law parameters:

$$u(z) = \frac{u_*}{\kappa} \ln\left(\frac{z - (h_p - h_s)}{z_0}\right) \quad (5)$$

κ is Von Karman's constant (0.41), h_s is the distance between the vegetation top and the surface layer virtual bed ($z_0 < h_s < h_p$) z_0 is the length scale for bed roughness of the surface layer (m) and u_* is the virtual bed shear .

Model 2 Righetti and Armanini (2002)

Righetti and Armanini (2002) developed an analytical two-layer model in order to describe a uniform flow condition. Fully submerged vegetation is considered as spherical shape. Differential equations are obtained by applying the double average (in time and space) of the Navier Stokes equations neglecting viscosity term. The integration of these equations from the free surface flow to the generic level z , leads to the following relationships for velocity distribution.

The vertical variation of the velocity in the vegetation layer is expressed as follows:

$$u(z) = \frac{2}{3} u_* \frac{h_p}{l} \left(\frac{z}{h_p}\right)^{2/3} \quad (6)$$

The only unknown parameter in this expression is the mixing length l . This parameter represents the scale integration process of turbulence and the turbulent dispersion. It was determined by experimental data.

In the surface layer, the velocity is defined by the following expression:

$$u(z) = \frac{u_*}{\kappa} \ln\left(1 + \frac{\kappa(z - (h - h_p))}{l_0}\right) \quad (7)$$

u_* is the shear velocity.

l_0 is the value of the mixing length at the interface level and in vegetated layer.

Model 3 Baptist et al. (2007)

Baptist et al. (2007) model is based on an analytical solution of the momentum balance of flow through and over vegetation. The Reynolds stress τ is determined using Boussinesq's eddy viscosity approach the mixing-length theory to define the eddy viscosity:

$$v_t(z) = C_p l u(z) \quad (8)$$

Where v_t is the coefficient C_p is the turbulent intensity, height – averaged over the vegetation height h_p and l is the mixing length. Baptist et al. (2007), proposed an empirical relationship for C_p and it is expressed as follow:

$$C_p l = \frac{1}{20} (h - h_p) \quad (9)$$

The expression of the velocity in the vegetation layer is given by the following equation:

$$u(z) = \sqrt{u_{v0}^2 + a_v \exp\left(\frac{z}{l}\right)} i \quad (10)$$

With:

$$u_{v0} = \sqrt{\frac{2g}{C_D m D}} \quad (11) \quad L = \sqrt{\frac{C_p l}{C_D m D}} \quad (12)$$

$$a_v = \frac{2Lg(h - h_p)}{C_p l \exp\left(\frac{h_p}{l}\right)} \quad (13)$$

L is the length scale (m) and a_v is integration constant.

For the surface layer, Prandtl's mixing length concept is adopted, and the velocity, follows a logarithmic profile:

$$u(z) = \frac{u_*}{\kappa} \ln\left(\frac{z - d}{z_0}\right) \quad (14)$$

Where d is the zero-plane displacement (m), which is located at distance from the bed inside the vegetation.

z_0 is the roughness length and u_* is the shear velocity.

Model 4 Yang and Choi (2010) model

Yang and Choi (2010), developed a simple relationships based on the two-layer approach, and on the momentum equation for the vegetation layer assuming uniform steady flow. Yang and Choi (2010) neglected the bottom shear stress, assuming that the stem drag is the dominant in the vegetation layer. Then, the velocity in this case is constant and it is given by the following expression:

$$U_1 = \sqrt{\frac{2ghi}{a C_D h_p}} \quad (15)$$

Where a is the density vegetation (m^{-1}).

In the surface layer, the velocity follows a logarithmic profile and depends on the density of vegetation:

$$u(z) = \frac{u_* C_\mu}{\kappa} \ln\left(\frac{z}{z_0}\right) + U_1 \quad (16)$$

In this model the roughness length equals to the vegetation height ($z_0 = h_p$).

C_μ is a constant depend on the vegetation density and it is given by the following equations:

$$\begin{cases} C_\mu = 1 & \text{for } a \leq 5 m^{-1} \\ C_\mu = 2 & \text{for } a > 5 m^{-1} \end{cases} \quad (17)$$

u_* is the interfacial shear velocity and it's determined by the following equation:

$$u_* = \sqrt{g(h - h_p) i} \quad (18)$$

III. Results and Discussion

The verification and the analyzing of these different analytic models for predicting the vertical velocity profiles are determined due to a comparison between the measured and simulated velocities.

The simulations were carried out by the Scilab software to reproduce the vertical profiles of measured velocities in a given section.

Table 1 shows, the experimental data flume available in the literature, concerning the free surface flow in presence of rigid and flexible, artificial and natural vegetation.

Experiments data of Shimizu and Tsujimoto (1994) and Lopez and Garcia (2001) were used to verify the capacity of these models in the case of rigid vegetation with high and low density. Experiments data of Tsujimoto et al.(1993) and Jarvela (2005) were used to verify the capacity of these models in the case of flexible vegetation (artificial and natural).

Table 1. Experiments data used for verification of the capacity of the three models in prediction of the velocity profile

Author(s)	Type	h(m)	h _p (m)	D(m)	m(m ²)	C _D	i
Shimizu and Tsujimoto (1994)	R	0.0936	0.046	0.0015	2500	1	0.02
Lopez and Garcia (2001)	R	0.335	0.12	0.0064	170	1.13	0.0036
Tsujimoto et al. (1993)	F	0.11	0.062	0.0062	1000	2	0.01
Jarvela (2005)	N	0.0936	0.046	0.0015	2500	1	0.02

R-Rigid vegetation, F- Flexible vegetation

The comparison between the measured and calculated velocity profiles was illustrated in the following figures:

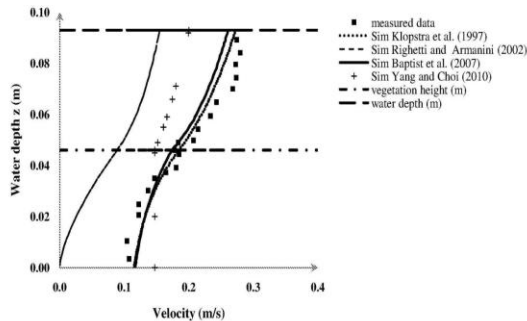


Figure. 2. Measured (Shimizu and Tsujimoto, 1994) and simulated flow velocities through rigid vegetation with high density.

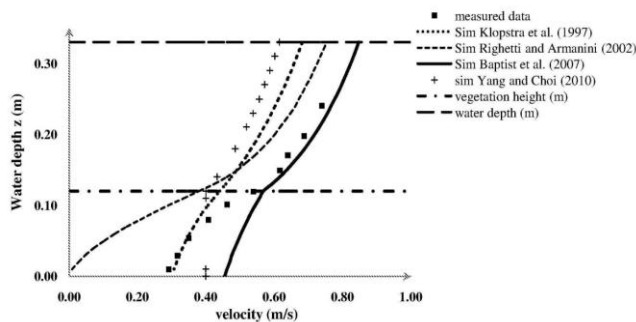


Figure. 3. Measured (Lopez and Garcia, 2001) and simulated flow velocities through rigid vegetation with low density.

Figure 2 shows simulation results in the case of rigid vegetation with high density. In this case, both Klopstra et al. (1997) model and Baptist et al. (2007) model indicates a performance in the prediction of the vertical velocity profile. In contrast, the models of Yang and Choi (2010) and Righetti and Armanini (2002) demonstrate an under estimation of the velocity. Yang and Choi model, assumed that the bed roughness height z_0 equals to the vegetation height, while, it was supposed smaller than the vegetation height in the models of Klopstra et al. (1997) and Baptist et al. (2007), and this could explain the underestimation observed in the surface layer. For Righetti and Armanini (2002) model, the difference between the measured profiles and the calculated is due to the mixing length l_0 . This parameter was determined by an empirical formula and validated only for a limited condition of a laboratory experiment. In the case of rigid vegetation with low density, the comparison between the simulated and measured profile indicates an inefficiency of whole models in the prediction of the vertical velocity (Figure 3). These analytic models neglect the effect of the bed roughness behind the vegetation roughness and this could explain the deviation observed between the measured and calculated profiles in this case because Bed roughness becomes more important, for sparse vegetation. All of these analytical models are set for rigid vegetation and it's questioned about their performance in calculating the behavior of flexible vegetation.

Figure 4 and 5 show the comparison between the measured and the calculated profiles through flexible vegetation.

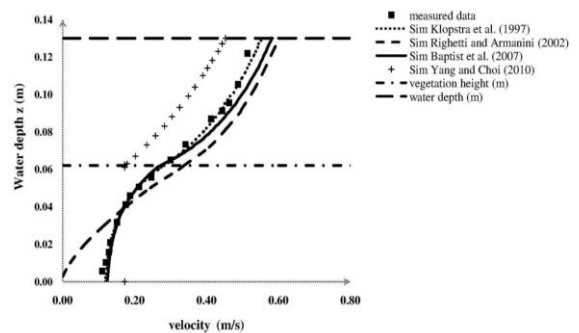


Figure 4. Measured (Tsujimoto et al., 1993) and simulated flow velocities through flexible vegetation.

In the case of flexible and artificial vegetation (Figure 4), calculated profiles by Klopstra et al. (1997) model and Baptist et al (2007) model were in good agreement with the measured profiles

The prediction of the velocity profile by these two models is also useful in the case of flexible vegetation. However, the simulations by Yang and Choi (2010) model, indicate an under estimation spatially in the vegetation layer and an over estimation was observed with the simulation by Righetti and Armanini (2002) model.

Figure 5 shows the performance of Baptist et al.(2007) model, for predicting the vertical velocity in the case of natural vegetation, only near the water surface, it is slightly over predicting the measured velocity profile.

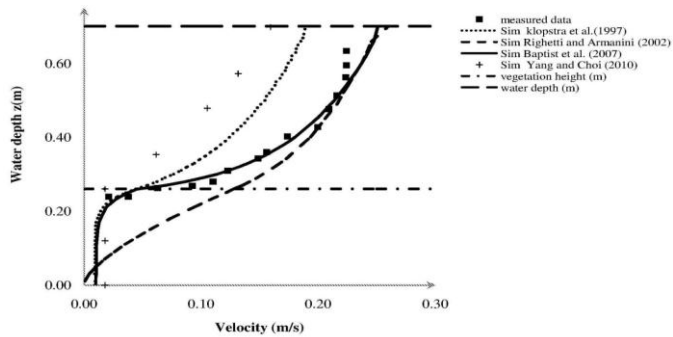


Figure 5. Measured (Jarvela, 2005) and simulated flow velocities through natural vegetation

For flexible vegetation, these models are less accurate than in the case of rigid vegetation. The flexibility of the vegetation is not even taken into account by these models. This could explain the deviations between the predicted and calculated velocity profiles.

IV. Conclusion

The results of these simulations for submerged vegetation show a performance of the two models of klopstra et al. (1997) and Baptist et al. (2007) in the prediction of the velocity profiles for rigid, flexible and natural vegetation with a high density. In contrast, the comparison between the measured and calculated profiles by Righetti and Armanini (2002) model and Yang and Choi (2010) model, indicates a discrepancy. The use of the proposal formula of the mixing length by Righetti and Armanini (2002) and the vegetation height as a roughness bed height could explain this offset deviation between the measured and calculated profiles. In perspective, we will consider tests for other models using more experimental data, to verify the prediction of the vertical velocity profiles and the mean velocity and select most suitable model that will be applied for a real cases, the river of Isere in France and Medjerda in Tunisia.

Acknowledgement

The author wishes to thank the Higher Education and Scientific Research Ministry in Tunisia for the financial support and the group of LTHER specially the RIVER team (Grenoble) for their cooperation.

- i. Baptist, M.J. Babovic, V. Rodríguez Uthurburu, J. Keijzer, M. Uittenbogaard R.E. Mynett, A. Verwey, A. 2007. On Inducing Equations for Vegetation Resistance. *J. Hydraul. Res.*, 45(4): 435-45.
- ii. Evangelos, K. 2012. The Impact of Vegetation on the Characteristics of the Flow in an Inclined Open Channel Using the Piv Method. *J. Wros.*, 1(1): 1-6.
- iii. Huai, W.X. Zeng, Y.H. Xu, Z.G. Yang, Z.H. 2009. Three-Layer Model for Vertical Velocity Distribution in Open Channel Flow with Submerged Rigid Vegetation. *Adv. Water Resour.*, 32(4): 487-492.
- iv. Huthoff, F. Augustijn, D. C. M. and Hulscher, S. J. M. H. 2007. Analytical Solution of the Depth-Averaged Flow Velocity in Case of Submerged Rigid Cylindrical Vegetation. *Water Resour. Res.*, 43(W06413): 1-10.
- v. Jarvela, J. 2005. Effect of Submerged Flexible Vegetation on Flow Structure and Resistance. *J. Hydrol.*, 307: 233-241.
- vi. Klopstra, D. Barneveld, H. J. van Noortwijk, J. M. and van Velzen, E. H. 1997. Analytical Model for Hydraulic Roughness of Submerged Vegetation. In: "Managing Water: Coping with Scarcity and Abundance". *Proc. of the 27th IAHR Congress, San Fransisco*, PP. 775-780.
- vii. Kubrak, E. Kubrak, J. and Rowinski, P.M. 2008. Vertical Velocity Distributions through and above Submerged, Flexible Vegetation. *Hydrolog. Sci. J.*, 53(4): 905-920.
- viii. López, F. and García, M.H. 2001. Mean Flow and Turbulence Structure of Open Channel Flow through Non-emergent Vegetation. *J. Hydraul. Eng.*, 127(5): 392-402.
- ix. Liu, C. and Shen, Y-M. 2008. Flow Structure and Sediment Transport with Impacts of Aquatic Vegetation. *J. Hydrodynamic.*, 20(4): 461-468.
- x. Righetti, M. Armanini, A. 2002. Flow Resistance in Open Channel Flows with Sparsely Distributed Bushes. *J. Hydrol.*, 269: 55-64.
- xi. Samani, J. M. V. and Mazaheri, M. 2009. An Analytical Model for Velocity Distribution in Transition Zone for Channel Flows over Inflexible Submerged Vegetation. *J. Agr. Sci. Tech.*, 11(1-2): 573-584.
- xii. Shimizu, Y. and Tsujimoto, T. 1994. Numerical Analysis of Turbulent Open-channel Flow over a Vegetation Layer Using a $k-\epsilon$ Turbulence Model. *J. Hydrosci. Hydraulic Eng.*, 11(2): 57-67.
- xiii. Stone, M.B. and Shen, H.T. 2002. Hydraulic Resistance of Flow in Channels with Cylindrical Roughness. *J. Hydraul. Eng.*, 128(5): 500-506.
- xiv. Tsujimoto, T. Okada, T. and Kontani, K. Turbulent structure of open-channel flow over flexible vegetation. *Progressive Report. 1993. Hydr. Lab., Kanazawa University, Japon.* pp.37-45.
- xv. Wu, w. and HE, Z. 2009. Effects of Vegetation on Flow Conveyance and Sediment Transport Capacity. *International Journal of Sediment Research.*, 24(3): 247-259.
- xvi. Yang, W. and Choi, S. 2010. A Two-layer Approach for Depth-limited Open-channel Flows with Submerged Vegetation. *J. Hydraul. Res.*, 48(4): 466-475.