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A Constrained Genetic Algorithm Based on Constraint Handling with KS Function and Grouping Penalty

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Abstract*：In order to overcome the limitation when using traditional genetic algorithm in solving constrained optimization problems, this paper presents a new method of constrain handling to solve the constrained optimization problems. Firstly, the method makes full use of the condensed characteristics of the KS function to transform multi-constrained optimization problem into a single constraint optimization problem. And then a group penalty method is adopted by genetic algorithm. Aggregate constraint reduces the solution scale effectively and improves the efficiency of searching for global optimization solution. The method of penalty in grouping is used to overcome the difficulty of penalty coefficient selection for general penalty function method. Several typical numerical experiments and engineering application show the performance and effectiveness of the proposed algorithm.

2. Key words: *KS function; grouping penalty; genetic algorithm; constraint handling*

1. Introduction

The engineering design problem usually can be transformed into constrained optimization problems. Most of optimization algorithms are efficient for unconstrained optimization problems or just with simple constraints. However, most engineering design problems may have many expensive constraints. To solve the constrained optimization problems, there exist some classical methods[1], such as penalty function method, Lagrangian multiplier method, Sequential Quadratic Programming(SQP) etc. But these gradient based optimization methods have some extra requirements, the objective and constrain function must be continuous and differentiable, and some even must be the high-order differentiable. Therefore the traditional optimization methods are not suitable for solving the discrete, discontinuous, derivative free optimization problems.

Since the classical optimization methods can not satisfy the growing requirements in calculation speed, convergence, sensitivity to initial value and so on, some constraint handling techniques are proposed for heuristic algorithms, such as evolutionary algorithms. Static Penalty[2] , dynamic Penalty[3] and adaptive penalty[4] methods are the most widespread approach because of its simplicity and ease of implementation. However, how to define the penalty term and penalty factors are the difficult aspects of penalty methods. Therefore some parameter-less[5] scheme have been proposed. These kinds of method usually treat the constraints and objective function separately, and need not penalty factors. But, when dealing with a large number of constraints, there is a high computational cost.

Since each penalty method has some limitations. A constrain handling combined with KS function and penalty in grouping are studied and applied to constrained genetic algorithm. KS function is used to transform constrains into a single continuous and differentiable constraint, and can reduce the solution scale effectively. In order to overcome the difficulty of defining the penalty term and penalty factors, a method of penalty in grouping is adopted.

2. Theory

2.1The aggregation properties of KS function

The KS function was first presented by G. Kreisselmeier and R. Steinhauser in 1979[6]. It is expressed as a real valued function is defined in n-dimensional Euclidean space, which can be differentiable envelope in the index space. It defined as:

$$KS(\rho,x)=\frac{1}{\rho}\ln\left(\sum_{j}^{l}\exp(\rho\cdot g_j(x))\right) \quad j=1,2,...,l \tag{1}$$

where ρ is a aggregation control parameter.

Let $$g_{\max}(x)=\max_j\{g_j(x)\} \tag{2}$$

The alternate form is adopted to reduce numerical difficulties caused by large values:

$$KS(\rho, x) = g_{\max}(x) + \frac{1}{\rho}\ln[\sum_j \exp(\rho \cdot (g_j(x) - g_{\max}(x)))] \qquad (3)$$

The following relationship can be easily deduced form the definition of KS function:

$$g_{\max}(x) \le KS \le g_{\max}(x) + \ln(N)/\rho \qquad (4)$$

From Eq(4), we can get:

$$\lim_{\rho \to \infty} KS(X, \rho) = \max(g_j(x)) \qquad (5)$$

As ρ approaches infinity, the KS function becomes equivalent to $g_{\max}(x)$, the maximum of the all the constraints[7].

As shown in Fig.1, The solid lines denote the constraints. The dotted line indicates that the KS functions of constraints corresponding to different parameters ρ. It shows that, by using the agglomeration features of KS function method, multiple constraints can be condensed into a constraint, which can improve the efficiency of solving optimization problems. And the KS function has the properties of continuous and differentiable and can approach the maximum of a set of function curves.

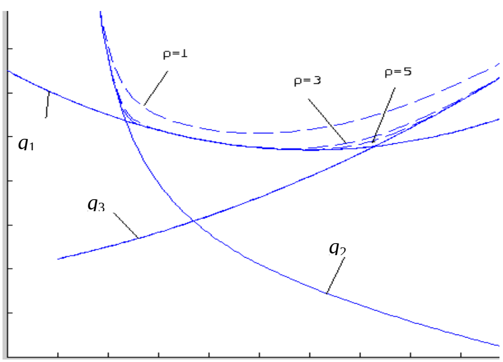

Fig. 1. The Characteristic of KS function

2.2The method of grouping penalty in GA

Penalty function method is the most commonly used method for handing constraints in genetic algorithm. How to set up a reasonable penalty coefficient is a big difficulty for the penalty function method to handle constraints. If the values of penalty coefficient are too large, it may be guarantee searching within feasible region, but it will reduce the exploration with infeasible region, result in losing some valuable information provided by the infeasible region, which may bring premature convergence to a local optimal solution. If the penalty coefficient is too small, then the penalty plays a smaller proportion in the objective function, it is difficult to find a feasible solution.

The basic idea of penalty by grouping is using two penalty coefficients P1 and P2 instead of one penalty as previous. The two penalty coefficients are related to the solutions of two different constraints satisfaction. This method has two advantages: Firstly, since the penalty coefficients are different, it will have different evolutionary trajectories of two groups of individuals. Because of the crossover and mutation in the two groups of individuals, it will increase the chance of escaping from the local optimal solution. Therefore it has better robustness than the general genetic algorithm. Secondly, in the constrained optimization problem, The global optimal solution usually located at the boundary between feasible and infeasible region. If a larger penalty coefficient (P1) and a lesser penalty coefficient (P2) are defined, convergence can be achieved from the boundary of feasible and infeasible region, Therefor, it will help us to quickly find the global optimal solution[8].

The proposed enhanced grouping penalty genetic algorithms with KS function (KS-GPGA) are presented as following:

Step1. Randomly generated 2m numbers of samples, where m is the population.

Step2. Divide the 2m numbers of individuals into two lists, each list is assigned with a penalty coefficient, and the constraints are transformed into one constraint by KS aggregating. Calculate the adaptive values of the 2m numbers of individuals.

Step3. Select the best m numbers of individuals from the 2m numbers of individuals by adaptive values.

Stp4. The m numbers of individuals are chosen to cross, mutation operator in genetic algorithm as initial point, Then create m numbers of new individuals.

Step5. Mix m numbers of new individuals with m numbers of parent individuals to form a new 2m numbers of individuals.

Step6. If not convergence, go to Step. 4.

3.Experimental and numerical analysis

There problems from Ref.[4] are solved to verify the performance of the new algorithm. And compared with group penalty genetic algorithms (GPGA).

Problem 1.

$$\min\ f(x)=5\sum_{i=1}^{4}x_i-5\sum_{i=1}^{4}x_i^2-\sum_{i=5}^{13}x_i$$

$$\begin{aligned}
s.t.\ & g_1(x)=2x_1+2x_2+x_{10}+x_{11}-10\le 0\\
& g_2(x)=2x_1+2x_3+x_{10}+x_{12}-10\le 0\\
& g_3(x)=2x_2+2x_3+x_{11}+x_{12}-10\le 0\\
& g_4(x)=-8x_1+x_{10}\le 0\\
& g_5(x)=-8x_2+x_{11}\le 0\\
& g_6(x)=-8x_3+x_{12}\le 0\\
& g_7(x)=-2x_4-x_5+x_{10}\le 0\\
& g_8(x)=-2x_6-x_7+x_{11}\le 0\\
& g_9(x)=-2x_8-x_9+x_{12}\le 0\\
& 0\le x_i\le 1(i=1,\ldots,9)\\
& 0\le x_i\le 100(i=10,11,12)\\
& 0\le x_{13}\le 1
\end{aligned}$$

The optimum of Problem 1 is:

$$x^*=[1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 3,\ 3,\ 3,\ 1]$$
$$f(x^*)=-15$$

Problem 2.

$$\min\ f(x)=(x_1-10)^3+(x_2-20)^3$$

$$\begin{aligned}
s.t.\ & g_1(x)=-(x_1-5)^2-(x_2-5)^2+100\le 0\\
& g_2(x)=(x_1-6)^2+(x_2-5)^2-82.81\le 0\\
& 13\le x_1\le 100,\quad 0\le x_2\le 100
\end{aligned}$$

The optimum of Problem 2 is:

$$x^*=[14.095,\quad 0.84296]$$
$$f(x^*)=-6961.81388$$

Problem 3.

$$\max\ f(x)=\frac{\sin^3(2\pi x_1)\sin(2\pi x_2)}{x_1^3(x_1+x_2)}$$

$$\begin{aligned}
s.t.\ & g_1(x)=x_1^2-x_2+1\le 0\\
& g_2(x)=1-x_1+(x_2-4)^2\le 0\\
& 0\le x_1\le 10, 0\le x_2\le 10
\end{aligned}$$

The optimum of Problem 3 is:

$$x^*=[1.2279713,\quad 4.2453733]$$
$$f(x^*)=0.095825$$

Numerical experiments conditions are defined as flowing: The initial population is 70. The number of iterations is 150. The probability of crossover is 0.2. The probability of mutation is 0.5. And every test problem runs 20 times independently under the same conditions, and recording the best value, the average value, the worst value and the standard deviation. A comparison of different results by genetic algorithm with the KS-GPGA is discussed.

From the table 1, we can see that for these problems, two kinds of methods can get good approximate optima. And it shows that the results obtained by the approach presented in this paper is better, whether the best value, average value, or the worst value, and the results are more close to the optimal solutions. The standard deviations of KS-GPGA are smaller than GPGA. The standard deviation shows the variation of results obtained by different solving. The smaller the standard deviation, the more concentrated results can be achieved, and it shows higher reliability of the algorithm. Therefore, the KS-GPGA show more advantages in solving constrained optimization problems.

In the Table 2, the method in this paper is compared with some methods in the literature researched before. In these methods, the Stochastic ranking method[5] can get better results. But this algorithm has great randomness. For some testing function, a feasible solution achieved after dozens of times solving. Therefore the stability should be taken into consideration. The test results show that the KS-GPGA can find the feasible optima solution at each run.

4. Engineering application

In order to further illustrate the effectiveness of this method, two engineering application are solved by KS-GPGA, and compared with existing methods.

Application 1. Tension/compression string structure design problem [15].

This problem consists of minimizing the weight of a tension/compression spring (shown as Fig.2) subject to constraints on minimum deflection, shear stress, surge frequency, and limits on outside diameter. The design variables are the mean coil diameter D(x_2); the wire diameter d(x_1) and the number of active coils N(x_3). Formally, the problem can be expressed as fowling:

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$$\min\ f(x) = (x_3+2)x_2x_1^2$$

$$s.t.\ g_1(x) = 1 - \frac{x_2^2x_3}{71785x_1^4} \le 0$$

$$g_2(x) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} - 1 \le 0$$

$$g_3(x) = 1 - \frac{140.45x_1}{x_2^2x_3} \le 0$$

$$g_4(x) = \frac{x_2+x_1}{1.5} - 1 \le 0$$

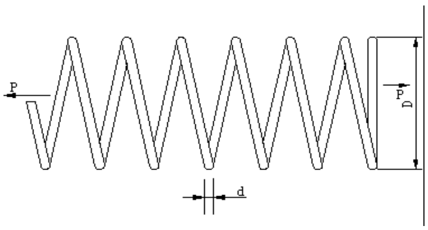

Fig.2. Tension/compression string structure design problem

This problem has been solved as a benchmark for constrained optimization by exist methods. Table 3 shows the results of KS-GPGA compared with existing methods. After 20 times of runs, the mean value achieved by KS-GPGA is better than others.

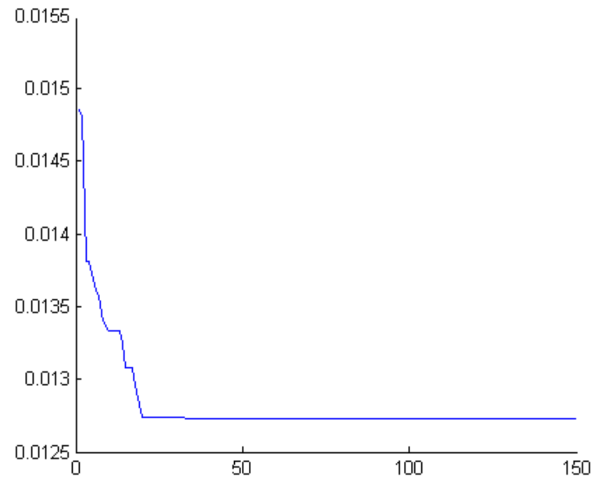

Fig.3. Convergence process of solving tension/compression string problem by KS-GPGA

Application 2. Design of a Pressure Vessel[15].

The pressure vessel problem is to design a compressed air storage tank with a working pressure of 3000 psi and a minimum volume of 750 ft^3. The schematic of a pressure vessel is shown in Fig. 43. The cylindrical pressure vessel is capped at both ends by hemispherical heads. Using a rolled steel plate, the shell is to be made in two halves that are joined by two longitudinal welds to form a cylinder. Each head is forged and then welded to the shell.

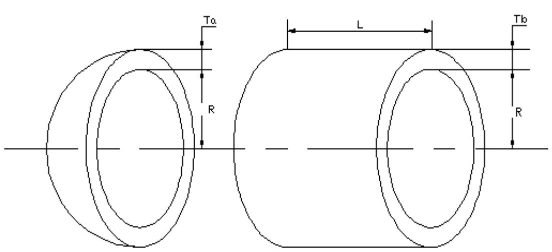

Fig. 4 Pressure vessel

Let the design variables be denoted by the vector: $X = [x_1,\ x_2,\ x_3,\ x_4] = [Th,\ Ts,\ R,\ L]$,where Th is the spherical head thickness, Ts is the shell thickness, R and L are the radius and length of the shell, respectively. The objective function is to reduce the combined costs of materials, forming, and welding of the pressure vessel. The mathematical model of the problem is expressed as follows:

$$\left.\begin{array}{ll} min & f(X) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1611x_1^2x_4 + 19.84x_1^2x_3 \\ s.t. & g_1(X) = 0.0193x_3 - x_1 \le 0 \\ & g_2(X) = 0.00954x_3 - x_2 \le 0 \\ & g_3(X) = 750\times1728 - \pi x_3^2x_4 - 4\pi x_3^3/3 \le 0 \\ & x_1 \in (0.1,\quad 99),\ x_2 \in (0.1,\quad 99) \\ & x_3 \in (10,\quad 200),\ x_4 \in (10,\quad 200) \end{array}\right\}$$

Where the design variables x_3 and x_4 are continuous and x_1, x_2 are integer multiples of 0.0625.

Table 4 shows the results of KS-GPGA compared with existing methods. After 20 times of runs, the mean value achieved by KS-GPGA is also better than others. The convergence process of solving pressure vessel design problem by KS-GPGA is illustrated as Fig.5

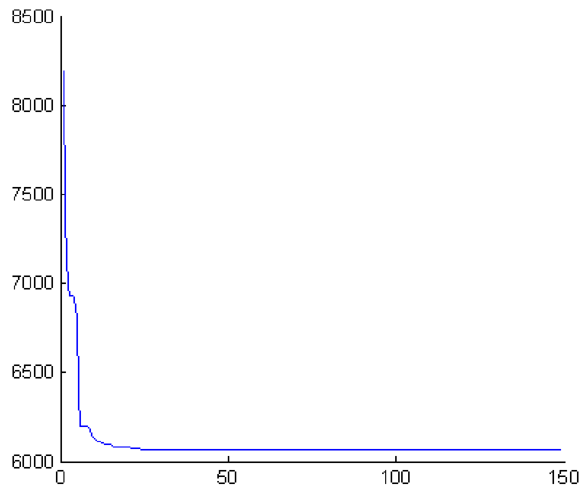

Fig.5. Convergence process of solving pressure vessel problem by KS-GPGA

5.Conclusions

Because of each constraint handling methods may Only suitable for a certain class of optimization problems. This paper presents a method KS-GPGA merging KS aggregate constraint function and grouping penalty method in genetic algorithm to solve constrained optimization problems. After the aggregate constraints with KS function, the constrained optimization problem is transformed to a single constraint optimization problem. Because the KS function has the properties of continuous and differentiable and can approach the maximum of a set of constraints, the simplified constraint is helpful to improve the computational efficiency. Method of grouping penalty is used to overcome the difficulty of penalty method to select the penalty factors. Several typical numerical experiments and engineering application show the reliability and effectiveness of the proposed algorithm.

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Table 1. Comparison of numerical results by two methods

Problem	Optimal value	GPGA				KS+GPGA			
		best	mean	worst	St.dev	best	mean	worst	St.dev
1	-15	-14.999	-14.900	-13.034	0.4395	-15.000	-14.999	-14.998	0.0006
2	-6961.81388	-6961.8	-6961.7	-6960.7	0.2428	-6961.8	-6961.7	-6961.6	0.2406
3	0.095825	0.095825	0.095825	0.095825	8.3586E-09	0.095825	0.095825	0.095825	9.2189E-09

Table 2. Comparison of the best value for the test functions of several methods

Problem	Optimal value	Koziel and Michalewiez [9]	Runarsson and Yao [10]	Tessema and Yen [11]	Deb [12]	Farmani and Wright [13]	Venkatraman and Yen [14]	KS+GPGA
1	-15.0000	-14.7860	-15.0000	-15.0000	-15.0000	-15.0000	-14.9999	-15.0000
2	-6961.81388	-6952.110	-6961.814	-6961.046	NA	-6961.800	-6961.179	-6961.8000
3	0.095825	0.095825	0.095825	0.095825	NA	0.095825	0.095825	0.095825

Table 3. Result comparison of the tension/compression string problem

Methods	$x_1(d)$	$x_2(D)$	$x_3(N)$	$f(x)$
Arora[17]	0.053396	0.399180	9.185400	0.0127303
Coello[16]	0.051989	0.363965	10.890522	0.012681
M-5[18]	0.05000	0.315900	14.25000	0.01283343
OPTDYN[18]	0.0644	0.7488	2.9597	0.01540256
KS+GPGA	0.0528	0.3482	9.8456	0.01268

Table 4. Result comparison of the pressure vessel design problem

Methods	$x_1(T_s)$	$x_2(T_h)$	$x_3(R)$	$x_4(L)$	$f(x)$
CPSO[21]	0.8125	0.4375	42.0912	176.7460	6061.0777
ADPSO[21]	0.8125	0.4375	42.0985	176.6370	6059.7143
SCPSO[22]	0.8125	0.4375	42.0984	176.6366	6059.7143
CAL-PSO[20]	0.8125	0.4375	42.0863	176.3824	6051.7504
ICOGA[19]	0.8125	0.4375	42.0924	176.6028	6057.9092
KS+GPGA	0.8441	0.4173	43.7332	157.3806	6008.4450