

## 3-Equitable Prime Cordial Labeling of Some Graphs

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**Abstract:** In this paper we have investigated the 3-equitable prime cordial labeling behavior of cycle with one chord, Twin chord, and split graph G.

**Keywords:** 3-Equitable Prime cordial Labeling, Cycle with one chord, Cycle with twin chord, and Split Graph. AMS Subject Classification: 05C78

### 1. Introduction

We begin with simple, finite, connected and undirected graph  $G = (V(G), E(G))$  with  $p$  vertices and  $q$  edges. For standard terminology and notations we follow Gross and Yellen [1]. We will provide brief summary of definitions and other information which are necessary for the present investigations.

**Definition 1.1** If the vertices are assigned values subject to certain condition(s) then it is known as *graph labeling*.

Any graph labeling will have the following three common characteristics:

- 1) A set of numbers from which vertex labels are chosen;
- 2) A rule that assigns a value to each edge;
- 3) A condition that this value has to satisfy.

**Definition 1.2** A ternary vertex labeling of a graph G is called a 3-equitable labeling if  $|v_f(i) - v_f(j)| \leq 1$  and  $|e_f(i) - e_f(j)| \leq 1$  for all  $0 \leq i, j \leq 2$ . A graph which admits 3-equitable prime cordial labeling is called a 3-equitable Graph.

**Definition 1.3** A 3-equitable prime cordial labeling of a graph G with a vertex set  $V(G)$  is a bijection  $f : V(G) \rightarrow \{1, 2, \dots, n\}$  and defined by

$$f(e = uv) = \begin{cases} 1, & \text{if } \gcd(f(u), f(v)) = 1 \text{ and} \\ & \gcd(f(u) + f(v), f(u) - f(v)) = 1 \\ 2, & \text{if } \gcd(f(u), f(v)) = 1 \text{ and} \\ & \gcd(f(u) + f(v), f(u) - f(v)) = 2 \\ 0, & \text{otherwise} \end{cases}$$

and  $|e_f(i) - e_f(j)| \leq 1$  for all  $0 \leq i, j \leq 2$ . A graph which admits 3-equitable prime cordial labeling is called a 3-equitable Graph.

**Definition 1.4** A chord of a cycle  $C_n$  is an edge joining two non-adjacent vertices of  $C_n$ .

**Definition 1.5** Two chords of a cycle  $C_n$  ( $n \geq 5$ ) are said to be twin chords if they form a triangle with an edge of the cycle  $C_n$ . For positive integer  $n$  and  $k$  with  $3 \leq k \leq n - 2$ ,  $c_{n,k}$  is the graph consisting of cycle  $C_n$  with a pair of twin chords with which the edges of  $C_n$  form cycles  $C_k, C_3$  and  $C_{n+1-k}$  without chords.

**Definition 1.6** For a graph G the split graph is obtained by adding to each vertex  $v$  to a new vertex  $v'$  is adjacent to every vertex that is adjacent to  $v$  in G the resultant graph is denoted as  $spl(G)$ .

The concept of prime cordial labeling was introduced by Sundaram et al.[10] and in the same paper they investigate several results on prime cordial labeling. Vaidya and Vihol [11] have also discussed prime cordial labeling in the context of graph operations. Murugesan[2] have introduce the concept of 3-equitable prime cordial labeling and proved that cycle  $C_n$ , Path  $P_n$ , Star graph and complete graph are 3-equitable prime cordial labeling. As 3-equitable prime cordial labeling is extension of prime cordial labeling and many papers published on it.

### 2. Main Results

**Theorem 2.1** Cycle  $C_n$  with one chord is 3-equitable prime cordial except  $n \equiv 2 \pmod{6}$ .

**Proof:** Let G be the cycle  $C_n$  with one chord. Let  $u_1, u_2, \dots, u_n$  be the vertices of cycle  $C_n$  and let  $e = u_1u_3$  be the chord of cycle  $C_n$ . We define the labeling  $f : V(G) \rightarrow \{1, 2, \dots, n\}$  as follows:

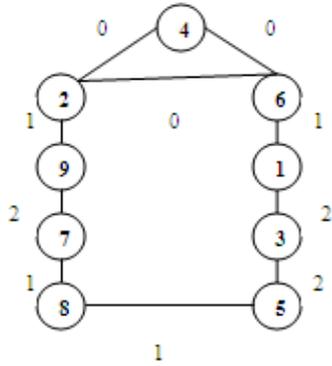
**Case 1: n is even.**

Put  $f(u_1) = 1, f(u_2) = 3, f(u_3) = 5$  and  $f(u_n) = n$ . Label the next three vertices (i.e.  $u_4, u_5, u_6$ ) by consecutive even numbers (i.e. 2,4,6 respectively) again next three vertices (i.e.  $u_7, u_8, u_9$ ) by consecutive odd numbers (i.e. 7,9,11 respectively) and repeat this process until all the vertices get labeled.

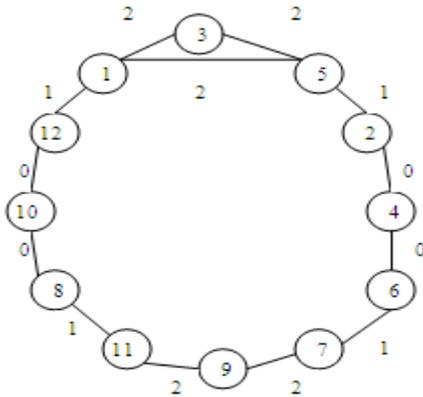
**Case 2: n is odd.**

Put  $f(u_1) = 2, f(u_2) = 4, f(u_3) = 6$  and  $f(u_n) = n$ . Label the next three vertices (i.e.  $u_4, u_5, u_6$ ) by consecutive odd numbers (i.e. 1,3,5 respectively) again next three vertices (i.e.  $u_7, u_8, u_9$ ) by consecutive even numbers (i.e. 8,10,12 respectively). Repeat the process until all the vertices get labeled. If there does not exist three even numbers or odd numbers then according to existence of number give the label. For better understanding see illustration 2.2 Figure 1.

**Illustration 2.2** The 3-equitable prime cordial labeling of cycle  $C_9$  (odd) and  $C_{12}$  (even) with one chord is shown in below **Figure 1** and **Figure 2** respectively.



**Figure 1:** 3-equitable prime cordial labeling of  $C_9$



**Figure 2:** 3-equitable prime cordial labeling of  $C_{12}$

**Theorem 2.3:** Cycle  $C_n$  with twin chords is 3-equitable prime cordial labeling except  $n \equiv 4 \pmod{6}$ .

**Proof:** Let  $G$  be the cycle  $C_n$  with twin chords. Let  $u_1, u_2, \dots, u_n$  be the vertices of cycle  $C_n$  and let  $e_1 = u_1u_3$  and  $e_2 = u_1u_4$  be the chords of cycle  $C_n$ . We define the labeling  $f : V(G) \rightarrow \{1, 2, \dots, n\}$  as follows:

**Case 1:**  $n \equiv 0 \pmod{6}$

Put  $f(u_1) = 2, f(u_2) = 4, f(u_3) = 6$ . Label the next three vertices ( i.e.  $u_4, u_5, u_6$  ) by consecutive odd numbers ( i.e. 1,3,5 respectively ) again next three vertices (i.e.  $u_7, u_8, u_9$  ) by consecutive even numbers ( i.e. 8,10,12 respectively). Repeat the process until all the vertices get labeled.

**Case 2:**  $n \equiv 1 \pmod{6}$

Put  $f(u_1) = 2, f(u_2) = 4, f(u_3) = 6$  and  $f(u_n) = n$ . Label the next three vertices (i.e.  $u_4, u_5, u_6$  ) by consecutive odd numbers (i.e. 1,3,5 respectively) again next three vertices (i.e.  $u_7, u_8, u_9$  ) by consecutive even numbers ( i.e. 8,10,12 respectively) and repeat this process until all the vertices get labeled.

**Case 3:**  $n \equiv 2 \pmod{6}$

Put  $f(u_1) = 1, f(u_2) = 3, f(u_3) = 5$  and  $f(u_n) = n - 1$  and  $f(u_{n-1}) = n$ . Label the next three vertices (i.e.  $u_4, u_5, u_6$  ) by

consecutive even numbers (i.e. 2,4,6 respectively) again next three vertices (i.e.  $u_7, u_8, u_9$  ) by consecutive odd numbers ( i.e. 7,9,11 respectively) and repeat this process until all the vertices get labeled.

**Case 4:**  $n \equiv 3 \pmod{6}$

Put  $f(u_1) = 2, f(u_2) = 4, f(u_3) = 6, f(u_n) = n - 1$  and  $f(u_{n-1}) = n, f(u_{n-2}) = n - 2$ . Label the next three vertices (i.e.  $u_4, u_5, u_6$  ) by consecutive odd numbers (i.e. 1,3,5 respectively) again next three vertices (i.e.  $u_7, u_8, u_9$  ) by consecutive even numbers ( i.e. 8,10,12 respectively) and repeat this process until all the vertices get labeled.

**Case 5:**  $n \equiv 5 \pmod{6}$

Put  $f(u_1) = 2, f(u_2) = 4, f(u_3) = 6,$

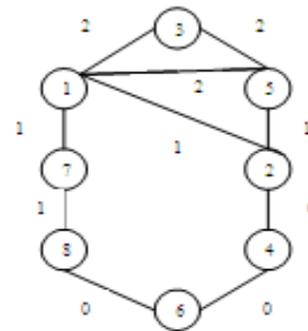
$f(u_n) = n, f(u_{n-1}) = n - 2,$

$f(u_{n-2}) = n - 4, f(u_{n-3}) = n - 1$  and  $f(u_{n-4}) = n - 3.$

Label the next three vertices (i.e.  $u_4, u_5, u_6$  ) by consecutive odd numbers (i.e. 1,3,5 respectively) again next three vertices (i.e.  $u_7, u_8, u_9$  ) by consecutive even numbers ( i.e. 8,10,12 respectively) and repeat this process until all the vertices get labeled.

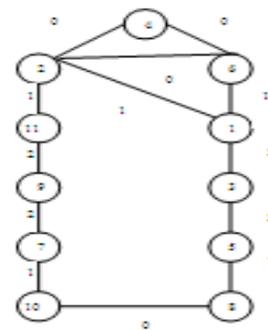
**Illustration 2.4** For better understanding of above defined labeling pattern, let us consider few examples.

Example 1: Cycle  $C_8$  with twin chord is 3-equitable prime cordial graph.



**Figure 3**

Example 2: Cycle  $C_{11}$  with twin chord is 3-equitable prime cordial graph.



**Figure 4**

**Theorem 2.5:** Split Graph  $spl(K_{1,n})$  is 3-equitable prime cordial graph.

**Proof:** Let  $v_1, v_2, \dots, v_n$  be the pendent vertices  $v$  be the apex vertex of  $K_{1,n}$  and  $u_1, u_2, \dots, u_n$  are the vertices corresponding to  $v_1, v_2, \dots, v_n$  in  $spl(K_{1,n})$ . Denoting  $spl(K_{1,n}) = G$  then

$|V(G)| = 2n + 2$  and  $|E(G)| = 3n$  to define  $f : V(G) \rightarrow \{1, 2, 3, \dots, 2n + 2\}$ . We consider following labeling pattern

$$f(v) = 1$$

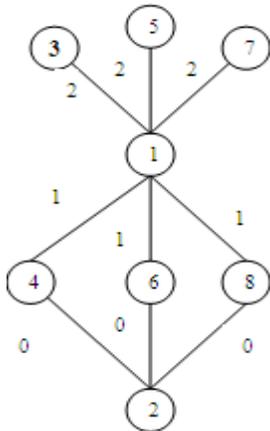
$$f(u) = 2$$

$$f(v_i) = 2i + 2, 1 \leq i \leq \lceil n/2 \rceil + 1$$

$$f(u) = f(v_i) - 1, 1 \leq i \leq \lceil n/2 \rceil + 1$$

In view of above labeling pattern defined above we have  $|e_f(i) - e_f(j)| \leq 1$ . Hence  $spl(K_{1,n})$  is 3-equitable prime cordial graph.

**Illustration 2.6** The 3-equitable prime cordial labeling of split graph  $spl(K_{1,n})$  is shown below:



**Figure 5.6** - 3-equitable prime cordial labeling of  $spl(K_{1,n})$ .

### 3. Conclusion

We proved here some new 3-equitable graph. To investigate similar results for other family as well in the context of different graph labeling problems is an open research area.

### 4. Acknowledgement

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