Effect of Rotor Solidity on the Tip Losses from Wind Turbine Rotor Blades

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Abstract: This paper discusses about the effect of rotor solidity on the tip loss occurring from blades of a turbine. The rotation of 3 bladed turbines commonly produces helical wake behind the rotor and subjected to several types of losses. One of the factors involved is loss arising from the blade tip and hub root. The performance of a wind turbine is also dependent upon the no of blades and rotor solidity which is function of blade chord. The number of blades increases the rotor solidity as well as tip loss as predicted by Prandtl. Aspects related to aerodynamic theory that proved power coefficient and tip losses are influenced by the solidity parameter. Comparisons of power coefficient using the different rotor lift to drag ratios are evaluated in order to verify the turbine efficiency range at different tip speed ratios.

Keywords: Inflow angle, Solidity, Tip loss factor, Airfoil, Thrust, Momentum, Induction factor

Nomenclature& Acronyms

BEM – Blade Element Momentum
TSR – Tip Speed Ratio,
L/D – Lift to drag ratio.
α - Angle of Attack, deg
β – Blade twist, deg
σ – Rotor solidity
c – Blade chord, m
r/R - Normalized blade span.
λ – Tip speed ratio.

1. Introduction

Wind turbines are emerging as the extensive source of producing electric power and are considered one of the efficient and clean methods. Turbines during operation undergo several rotations typically of order of millions and subjected to forces acting on structural components. The efficiency of turbines not only depends upon parameters such as thrust coefficient, torque and power coefficients but also the factors such as the rotor solidity, tip loss and hub loss from the turbine blades. The prandtl tip loss factor considers the effect of no of blades as well as the axial induction factor which accounts for the efficiency from a turbine. The power coefficient on the turbine have been quantified using analytical methods such as the BEM theory which combines axial momentum and blade element in order to evaluate the loads and efficiency, however the most widely acclaimed theories [i] that were developed in the past are Glauert momentum and axial momentum, vortex cylinder which predict the thrust and power coefficients and its variation with axial induction factor, and resulting torque produced from the turbine.

2. Rotor Solidity& L/D ratio

The efficiency of rotor blades of a turbine is dependent upon the aero foil profiles used for the blade structure. An airfoil has typically the following important structural parameters which affect the efficiency of a turbine blade.

1. Chord
2. Thickness, (span wise and chord wise)
3. Camber
4. Solidity

These properties change the lift coefficient across the blade span which is responsible for producing the useful torque required for a turbine to power. The efficiency of turbine depends upon the lift and drag coefficients generated by the blade during operation for a given wind speed regime. The solidity correlates the sectional blade chord along with no of blades at a given span wise station of blade. So, local rotor solidity is written in terms of blade chord and normalized blade span as

$$\sigma_l = \frac{Nc}{2\pi\frac{r}{R}} \qquad (1)$$

Where N – no of blades; c – local blade chord; r/R represents the normalized blade span. Fig 1 shows the airfoil section with force components

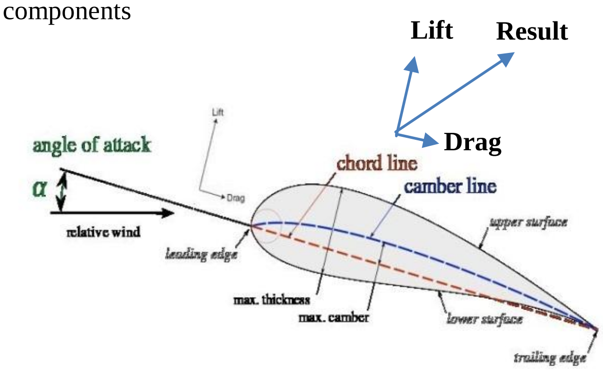

Fig 1: Airfoil nomenclature & forces on airfoil

The turbine extracts power from the wind due to above structural properties. In this case a blade length of 17m is considered whose structural parameters are listed in the table below

Sl.No	r/R [-]	Chord, [m]	Twist [deg]
1	0.2	1.085	15
2	0.25	1.045	12.1
3	0.3	1.005	9.5
4	0.35	0.965	7.6
5	0.4	0.925	6.1
6	0.45	0.885	4.9
7	0.5	0.845	3.9
8	0.55	0.805	3.1
9	0.6	0.765	2.4

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10	0.65	0.725	1.9
11	0.7	0.685	1.5
12	0.75	0.645	1.2
13	0.8	0.605	0.9
14	0.85	0.565	0.6
15	0.9	0.525	0.4
16	0.95	0.485	0.2
17	0.99	0.445	0

Table 1.Structural properties of rotor blade

The number of blades used in a turbine and chord lengths from the overall cross section of the blade affects the rotor efficiency. With increasing advances in the blade technology such as the winglets and trailing edge serrations or flaps used in aircraft industry, a higher lift coefficient are obtained and continue to contribute to improved energy yield from the turbine. The axial induction factors at the disc change the inflow angles and the lift forces on the blade. The tangential component of lift force contributes to the torque and this is observed to be lower values at the blade tips than other sections and hence tip losses are observed on the turbine blades due to changing airfoil profiles geometry.

3. Results & Discussion

The turbine performance is influenced by the aerodynamic lift coefficient, structural parameters and wake structure behind the rotor. The power coefficient of turbine has been evaluated using the several lift and drag coefficient ratios in order to analyze the change in its performance. The peak efficiency would reach when the un-deformed successive helical vortices are spaced equidistant in the turbine wake. The following figure shows the effect of lift to drag ratios (L/D) of turbine rotor or the aerodynamic efficiency of rotor on turbine performance. This magnitude of L/D ratios depend upon the type of airfoils chosen for blade construction. Typically thin airfoils either NACA 6x-xxx [iv] series are selected towards the outboard sections of blade which contribute to high lift while thick airfoils towards the inboard sections of blade. The lift produced by the rotor blade lies within a range of 30 – 80% of blade span. The thick airfoils together with the cylinder root are manufactured in such a way so as to provide high strength and significant lift and torque required for the blades to produce power.

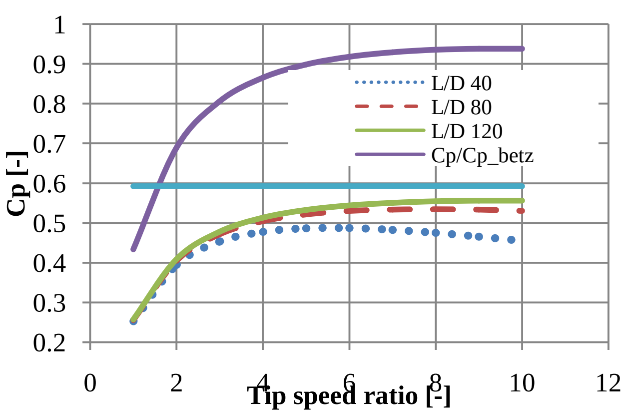

Fig 2. Power coefficient with varying aerodynamic efficiencies of rotor

According to BEM theory, the maximum power coefficient which can be obtained for a horizontal axis turbine is16/27 = ~ 59.3 %. This is known as the Betz limit. Here the power coefficient for a turbine containing multiple blades is calculated for various (L/D) ratios using the following procedure

- The inflow angle for each blade section is calculated as using the expression
  $\emptyset = \tan^{-1}\left[\frac{1}{\left(\frac{r}{R}\right)\lambda}\right]$ Which is checked for the condition when it is greater than zero, otherwise the inflow angle is set to zero. Where (r/R) is the normalized blade span, λ is the tip speed ratio of the turbine.
- The induction factor is then evaluated as function of tip speed ratio and the inflow at every blade section.
- Analytical method is used for the determination of blade profile section efficiency and tip loss factor for blade and summed up for a given tip speed ratio.
- The profile rotor efficiency is function of the L/D ratio chosen i.e. L/D = 40, 80 & 120 and expressed mathematically as

$$\eta = 1 - \frac{\lambda}{C_l/C_d} \qquad (2)$$

$$a = 4\lambda\mu^2\left[\frac{\sin\left[\frac{2}{3}\emptyset\right]^3}{\sin\left[\emptyset\right]^2}\right]d\mu \qquad (3)$$

- The tip loss factor is predominantly varies with the number of blades and the tip speed ratio of the turbine. It can be noted that the inflow angle for the blade section is dependent upon the local speed ratio however; the overall turbine efficiency is quantified using the section efficiency, tip and induction factors along the blade span of rotor.
- The ratio of turbine efficiency for the assumed aerodynamic ratios, to the maximum possible efficiency is interpreted in the fig 2 which considers the tip losses due to wake from the turbine as $C_p/C_{p,betz}$ This is an indicator of the relative turbine operating efficiency. [v]

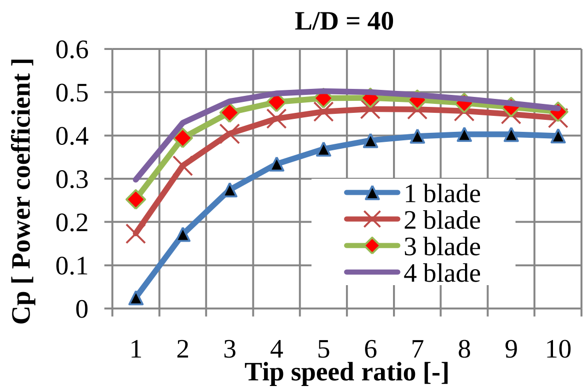

Fig 3. Power coefficient with varying no of blades, L/D = 40 including tip losses.

From the above fig 3 the turbine coefficient is calculated using the conservative ratio of L/D = 40 aerodynamic efficiency. In practice, the aerodynamic efficiency can vary up to L/D=120, the higher the L/D ratios tend to predict higher C_p for a turbine. However, it must be noted that a value exceeding the betz limit ~ 59.3 % indicates inaccurate profile assumptions for the blade of a turbine. It does not adhere to the traditional wind turbine rotor theories i.e. axial momentum and vortex cylinder. It can be expressed mathematically as

$f(\mu) = f_t(\mu).f_r(\mu)$ (4)

Where f – Combined root & tip loss factor

$f_r(\mu)$ - Root loss factor

$f_t(\mu)$ – Tip loss factor

μ – Normalized blade radius, r/R

μ_R – Normalized blade root radius,

N- No of blades

a – axial induction factor

‘λ – Tip speed ratio

$$f_r(\mu) = \frac{2}{\pi}\cos^{-1}\left[e^{-\left(\frac{N}{2}\right)(\mu-\mu_R)/\mu\sqrt{1+(\lambda\mu)^2/(1-a)^2}}\right] \quad (5)$$

$$f_t(\mu) = \frac{2}{\pi}\cos^{-1}\left[e^{-\left(\frac{N}{2}\right)(1-\mu)/\mu\sqrt{1+(\lambda\mu)^2/(1-a)^2}}\right] \quad (6)$$

The blade rotation causes the flow around the airfoil to produce predominant flow interactions along the axial, radial and tangential directions respectively. The air foil also experiences dynamic stall phenomenon when the blade angle of attack exceeds the stall angle, typically about 10-14 degrees. Due to the blade rotation, the vortices are produced in the tip of the blade as well as the root or hub section of the turbine. In this section, only the tip loss effect as derived by Prandtl is examined which contribute to the aerodynamic losses from the turbine blade [ii]. Further a comparison has been done in order to elucidate and quantify the aerodynamic loss at varying tip speed ratios along the blade span. It can be noted that the tip loss factor is considered significant importance for the turbine operation in a large wind farms since the flow field behind the turbine or the wake is distorted and generally expands downstream during the operation and results in the reduction of the wind speed experienced by the neighboring turbines located in the wind farm. This normally tends to lower the power production from the adjacent turbines also and increases turbulence in the flow field.

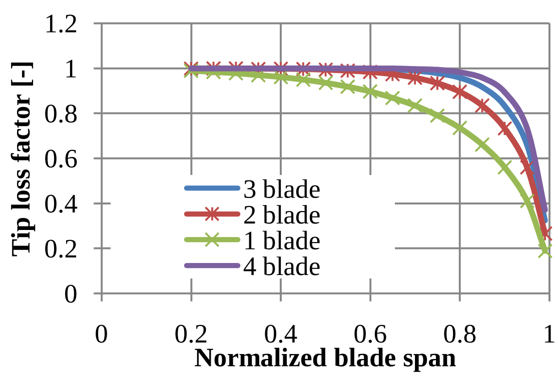

Fig 4. Prandtl Tip loss correction for multiple blades

Vortexes are shed from the tip of blade and root which form a series of helical vortex and run downstream axially from the center of disc [iii]. The root vortex is responsible for induced tangential velocity in wake flow of rotor.

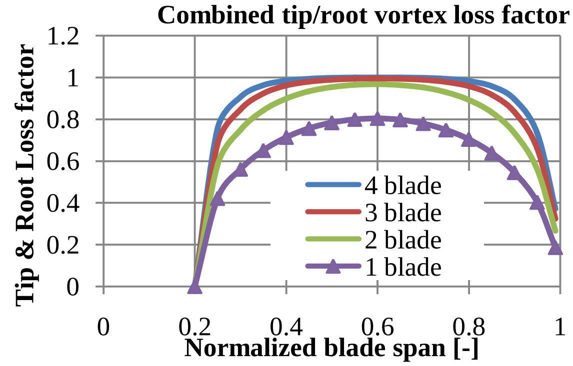

Fig 5. Prandtl tip loss correction combined with root or hub vortex loss

The combined effect of tip loss and root or hub losses acting on blade are shown in fig 5. The blade tip and hub losses are calculated from the 20 % of blade span, and it can be noted that maximum tip and hub losses are contributed around the 40-80% of blade span for different blade solidities. They reach low values towards the tip of blade and found to be lowest for a single blade turbine. The axial induction factor variation along the blade span is shown for the cases local to blade and azimuth direction of blade span which seem to diverge at the tip.

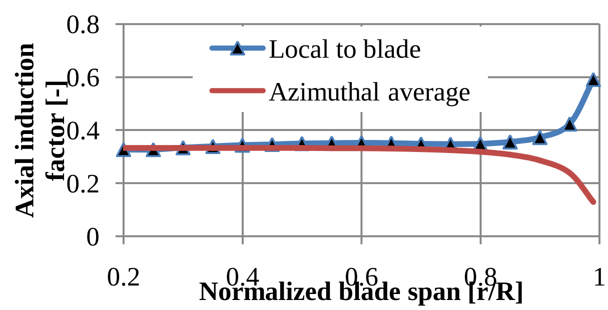

Fig 6. Axial induction factor variation local to blade & azimuth averaged

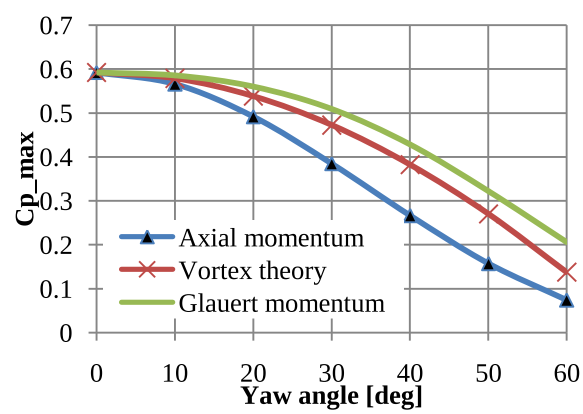

Fig 7. Maximum power coefficient validation using aerodynamic theories

The aerodynamic theories applicable to turbine operation are momentum theories in which, Glauert’s momentum theory predicts a higher thrust coefficient on turbine rotor, while vortex cylinder theory depicts the wake of turbine rotor as cylindrical vortex sheets emanating from the blade surface. Each of these theories predicts a different level of thrust and power coefficients. In fig 6 a comparison of power coefficients is done for varying yaw angle operation as function of axial induction factor and wake skew angle. As the yaw angle increases during the operation, the power coefficient also reduces significantly with increasing axial induction. For yawed rotor the wake is deflected to one side due to induced velocity which aligns orthogonal to wind direction flow and thrust coefficient continues to increase with axial induction factor. For non-yawed rotor the induced velocity at the disc is found to be half the value in the wake [ii] and thrust coefficient reaches maximum at a =0.5 after which momentum theory breaks down and continue to decrease with further increase in axial induction factor. .

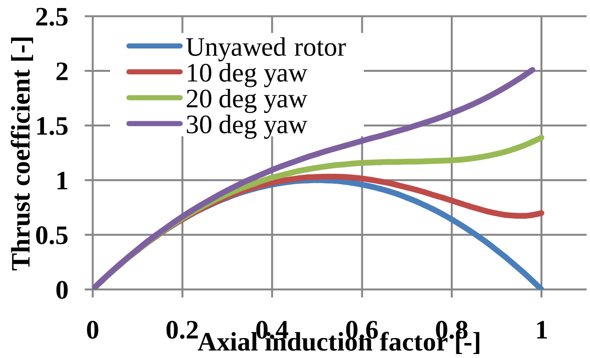

Fig 8 Illustration of thrust coefficient with yaw angle from glauert theory

Lifting line theory suggested by (Prandtl and Tietjens 1957) depicts helical wake vortex and produces downwash which is uniform along the blade span. This theory considers that normal induced velocities at rotor disc are important in predicting the inflow angles and angle of attack [iv] which often result in higher forces and thrust on the rotor blade. The power and thrust coefficient for yawed turbine rotor can be written as

$$C_p = 4a[\cos\gamma - a]^2 \quad (7)$$

$$C_{pmax} = \frac{16}{27}(\cos\gamma)^3 \quad (8)$$

$$C_p = 4a\left(\cos\gamma + \tan\frac{\varphi}{2}\sin\gamma - a\sec^2\frac{\varphi}{2}\right)(\cos\gamma - a) \quad (9)$$

$$\varphi = (0.6a + 1)\gamma \quad (10)$$

$$C_p = 4a(\cos\gamma - a)\sqrt{1 - a(2\cos\gamma - a)} \quad (11)$$

$$a = \frac{\cos\gamma}{3} \quad (12)$$

$$C_T = 4a\sqrt{1 - a(2\cos\gamma - a)} \quad (13)$$

Where γ is the yaw angle of rotor, a is the axial induction factor, φ is the wake skew angle. The axial momentum theory predicts the power coefficient better than the vortex cylinder and glauerts momentum theory. The prevalent wind direction at a site is measured using the wind direction sensor. This signal is sent to the main controller for further processing with referent data related to number of turns rotated by the nacelle of turbine. The main controller compares the desired yaw turns and stops the turbine from further yawing to prevent the excessive twist of the power cables which run from top head down to the transformer room at the base of the machine. The *equation 8* is the case for axial momentum theory while *equation 9* is according to vortex cylinder theory and *equation 11* represents the Glauerts momentum theory for yawed rotor conditions respectively.

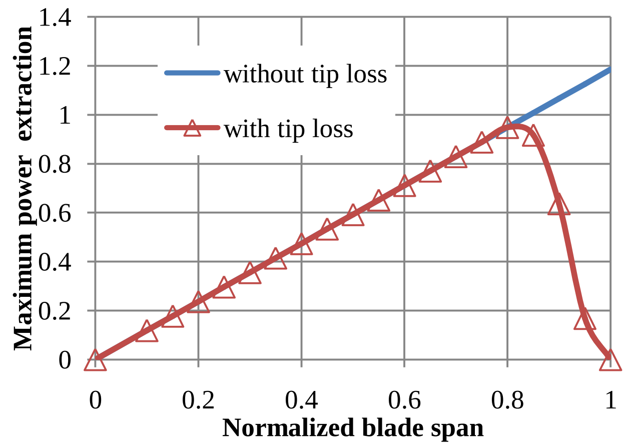

Fig 9.Comparison of maximum energy extraction along blade span with and without tip loss factors

4. Conclusions

Turbine operation for varying yaw conditions is validated for measuring the turbine efficiency by using vortex cylinder, axial momentum & glauert momentum theory. The effect of design parameters such as the lift to drag ratios of a rotor and tip loss on the turbine performance suggests that the blade rotation speed as well as the no of the blades impacts the efficiency in a wind turbine. Hence the wind farm design layout is critical to the optimal power production from wind farm which need to consider the influence of tip loss measured using the tip loss factor for wind turbines operating in large wind farms. The number of blades used in a turbine increases the turbine solidity and affects the magnitude of the tip loss which reaches maximum between the 40-80% blade span. The maximum power extraction from the blade increases towards the blade tip which is computed for with and without tip loss factor hence outline the importance of tip loss during the operation. The axial induction factor tends to increase towards the blade tip radial direction and decreases in azimuth directions respectively. The tip loss factor estimated by the prandtl theory is less compared to vortex cylinder theory. Higher values of lift to drag ratios result in higher efficiency of turbine. For a given lift to drag ratio i.e. aerodynamic efficiency of rotor, increasing the rotor solidity tend to increase the power extraction ability of turbine rotor.

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