

# Topology Optimization of Truss

Naresh D. Sonwane, K.N. Kadam

Applied Mechanics Department, Government College of Engineering Amravati, 444604, Maharashtra, India  
email: naresh.sonwane123@gmail.com

**Abstract :** *The optimal design of skeletal structure becomes imperative both from engineering and cost considerations in recent year. Total cost of the structure mainly depends on weight of the structure and weight of the structure is proportional to material distribution within the structure. The aim of study is to achieve minimum weight of the structure under nodal displacement, stress and stability constrains. This paper deals with linear static analysis and topology optimization of the skeletal structure with two noded line elements as a non linear problem. Optimization of 6 noded, 11 member 2D benchmark truss problem is carried out, in order to find out best distribution of material between the nodes after satisfying all constrains to achieve the minimum weight of the structure. Comparison of the displacement and weight with benchmark problem is studied. Optimization has performed with the help of HyperWork computer program tool and iteration obtained after topology optimization is discussed.*

Keywords Benchmark, Optimization, Topology, Truss

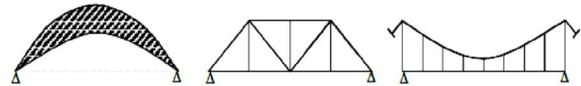
## Introduction

Structural optimization has become a valuable tool for engineers and designers in recent years. Although it has been applied for over fifty years, optimization in engineering has not been a commonly used design tool until high performance computing systems are widely available. Structures are becoming lighter, stronger, and cheaper as industry adopts higher forms of optimization.

Primary purpose of optimization is to find the best result to a problem with given set of circumstances. Computer-based optimization launched itself into the engineering industry due to the fact that the topic lends itself to numerical computation, which is the one task in which computers have superiority over humans. Programmers immediately began introducing new optimization methods such as nonlinear programming, unconstrained optimization, and multi-objective optimization.

The design of truss structures can be defined as one or any combination of three optimization problems, namely, size, topology and geometry (shape) optimization, as shown in Fig 1. Size (dimensioning) optimization is about searching for the optimal cross-sectional area of structural elements (bars), by assuming that the connectivity and nodal coordinates of the truss are fixed. Topology optimization is concerned with connectivity or the problem of to be or not to be of the elements between the nodes. And, finally, shape optimization is defined as determination of the optimum nodal coordinates, assuming that the topology is fixed.

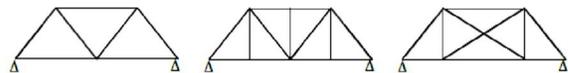
Choice of the principal idea:



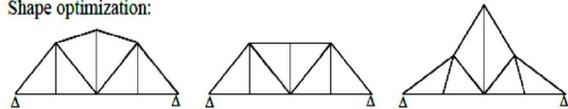
Choice of the material behavior:

Aluminum      Steel      Composites

Topology optimization:



Shape optimization:



Dimensioning:

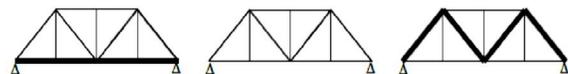


Figure 1. Classification of the Structural Optimization Task on Kind of Design Variable

Under topology optimization, the material density of each element should take a value of either 0 or 1, defining the element as being either void or solid, respectively. Optimization of a large number of discrete variables is computationally restricted. Therefore, representation of the material distribution problem in terms of continuous variables has to be used. With the density method, the material density of each element is directly used as the design variable, and varies continuously between 0 and 1; these represent the state of void and solid, respectively. Intermediate values of density represent fictitious material. The stiffness of the material is assumed to be linearly dependent on the density. Most of the work has been dedicated to the so-called maximum stiffness (or minimum compliance) formulations. However, since a few years different approaches have been proposed in terms of minimum weight with stress (and/or displacement) constraints. HyperWorks v.13 is a comprehensive simulation platform for rapid design exploration and decision-making. It consist modeling, analysis, optimization, visualization, reporting and collaborative simulation management.

Three different approaches of the stress constraints for the topology optimization of structures problem: the local approach, the global approach and the block aggregated approach presents and compare in [1]. Two phase hybrid method for the size and topology optimization of truss

structures is presented for optimization of 11 member 6 node truss [2]. Static analysis of truss using FE method coded program and source program (SAP2000) has performed. Optimization has been carried out with the help of GA and it is found that after the 65<sup>th</sup> iteration the weight of the roof truss got minimized up-to 54% for continuous design variable [3].

## 2. Formulation

CROD element of HyperWork element library has been used; it is 1D line element having only one degree of freedom which is displacement in axial direction. Therefore only axial stresses and axial displacement result after analysis. Equation (1) is the Stiffness matrix of truss element comprised of geometric and material property under analysis process. Generally force vector, material and geometrical property are known in stiffness matrix to find displacement vector. If displacements do not satisfy the feasible design, recursion process continues until optimization is achieved. Equation (2) is the objective function for minimization of weight. Equation (3) and (4) is constrains of displacement in x-axis and y-axis respectively. Stress constrains are represented by (5).

Stiffness matrix for static analysis of truss is

$$\begin{Bmatrix} f_i \\ f_j \end{Bmatrix} = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} \quad (1)$$

where,

- $f$  = axial force, kN;
- $A$  =Area of member, mm<sup>2</sup>;
- $E$  =Modulus of elasticity, N/mm<sup>2</sup>;
- $L$  =Length of member, m;
- $u$  =Nodal displacement, mm; and
- $i, j$  =1, 2, 3.....n, (n= no of members).

Formulation of topology optimization is base on stress and displacement approach,

Minimize weight,

$$\min (W) = \sum A_e L_e \rho_e \quad (2)$$

Subjected to,

$$u_k^{\min} \leq u_k \leq u_k^{\max} \quad (3)$$

$$v_k^{\min} \leq v_k \leq v_k^{\max} \quad (4)$$

$$\sigma_n^{\min} \leq \sigma_n \leq \sigma_n^{\max} \quad (5)$$

where,

- $A_e$  =Area of member, mm<sup>2</sup>;

$L_e$  =Length of member, m;

$\rho_e$  =Material density, kg/m<sup>3</sup>;

= Axial stress, N/mm<sup>2</sup>;

$u$  =Displacement in x-axis, mm;

$v$  =Displacement in y-axis, mm;

$k$  =1, 2, 3.....m, (m= no of nodes); and

$n$  =0, 1, 2, 3.....n, (n= no of element).

Formulation of topology optimization is based on minimize compliance and maximum volume,

$$\min_x f^T u(x) \quad (6)$$

Subjected to,

$$K(x) u(x) = f \quad (7)$$

$$0 \leq x \leq 1$$

Given that,

$$K(x) = \sum_i x_i K_i \quad (8)$$

Compliance minimization is given by (6),  $f$  is vector of applied load (7) and  $K$  is the element stiffness matrix associated with variable  $i$  (8),  $u(x)$  is displacement vector,

## 3. Numerical example

A 2D truss made up of 11 members is, as shown in Fig 2. Areas of all members are taken as 0.01935m<sup>2</sup>. Truss support two downward force of same magnitude given as 444.82 kN acting at node 3 and node 5. The allowable nodal displacement is 0.0502m. All members are subjected to an allowable axial stress of  $\pm 172.368$  MPa, modulus of elasticity of material is taken as  $E=68.95$  GPa and material density is  $\rho=2768$  kg/m<sup>3</sup>. HyperWorks v.13 tool is use to topology optimization.

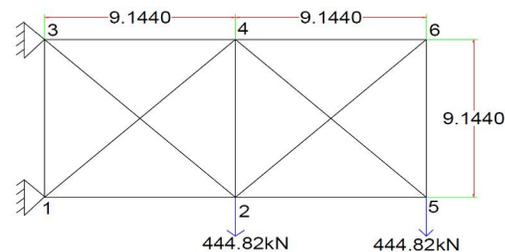


Figure 2. Benchmark Truss Problem

Static analysis of truss gives the displacements and elongations corresponding to nodes and members respectively are shown in Fig 3. In this figure dark blue coloured members are subjected to maximum elongation and minimum or zero elongation with red coloured members. Maximum deflection of 35.5mm is figure out at end node 5.

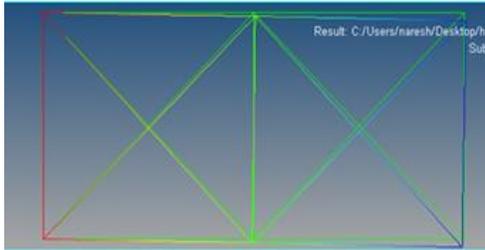


Figure 3. Displacement after Static Analysis

Topology optimization of truss provides the optimized topology after satisfying all constrains. Iteration of topology optimization is, as shown in Fig 4. Element density is distributed to satisfying displacement and stress constrains to minimize weight. Result for topology optimization obtained by HyperWork computer tool converged and provide final topology after 9<sup>th</sup> iteration it is reduced weight truss due to 5 members of truss are removed in optimization process.

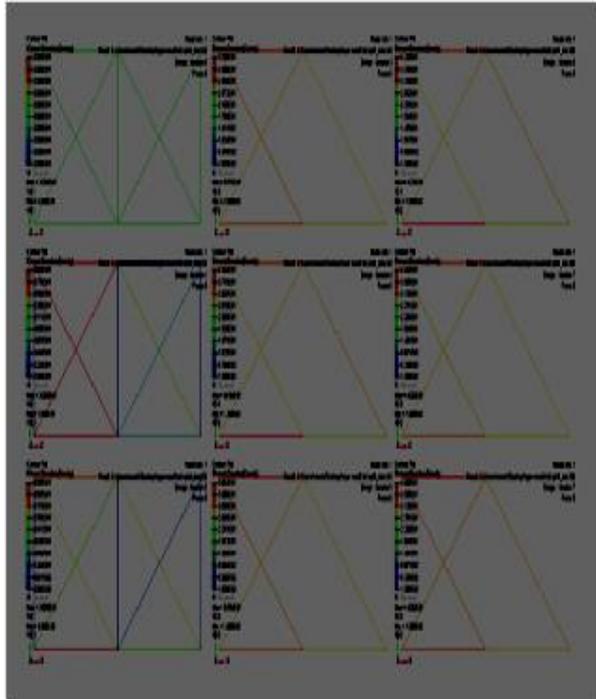


Figure 4. Iterative Result of Topology Optimization

Final optimised design by proposed analysis is, as shown in Fig 5A, it is similar to final optimised design obtained in [2] is, as shown in Fig 5B.

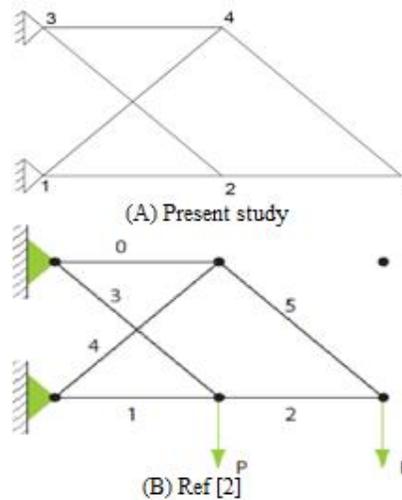


Figure 5. Comparison of Truss A and Truss B Topology

After optimization resulting topology further linear statically analyse to cross check the values of maximum displacement and maximum downward displacement.

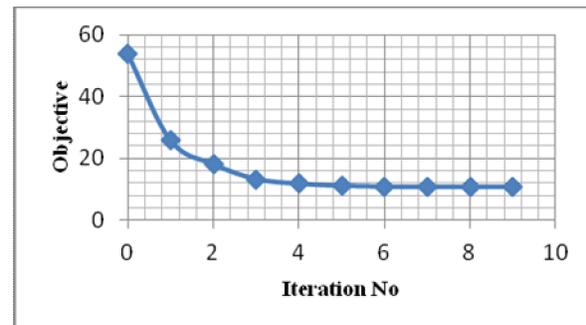


Figure 6. Convergence History

Convergence history of the optimization problems is, as shown in Fig 6.

Table 1. Comparisons of Joint Displacement and Weight

Parameter	Initial Truss	Optimized Truss	% Improvement
Weight (kg)	6062.983	3350.227	42.765
Max Joint Displacement (mm)	35.5	42.3	-

Initially calculated weight of truss is 6062.983kg and weight calculates after final design obtained due to optimization process is 3350.227kg, as shown in Table 1. The percentage change in weight after optimization is 42.765% of

initial weight and final weight is 57.232% of initial weight of truss.

Displacement obtain after static analysis of benchmark truss is 35.5mm and displacement of final design obtained by topology optimization is 42.3mm, as shown in Table 1 this values of displacement satisfy the constrains of displacement taken in the problem initially.

#### 4. Conclusions

The present study, an optimization technique for indeterminate truss has been studied using HyperWork software. An approach is proposed based on using element densities of the member as design variable and nodal displacements and stresses are constraints. The following conclusions are made. Objective function for minimizing the weight depends upon various parameters such as element density, area, node coordinate as design variable. Number of iterations to obtain the results is less it means convergence of result is achieved in less computational time. Weight of optimized truss is 57.232% of the benchmark truss structure. Linear static result produce by tool is similar to the results obtained by analytical method. The present work is a good contribution to use the structures to its maximum capacity and to enhance nation economy by reduced material consumptions.

#### REFERENCES

- i. J. Paris , F. Navarrina, I. Colominas, M. Casteleiro, "Improvements in the treatment of stress constraints in structural topology optimization problems", *Journal of Computational and Applied Mathematics* 234 (2010) 2231-2238.
- ii. A. Faramarzi, M.H. Afshar, "Application of cellular automata to size and topology optimization of truss structures", *Scientia Iranica A* (2012) 19 (3), 373–380.
- iii. Galawezh Saber, Nildem Tayşi, Ghaedan Hussein, "Analysis and Optimum Design of Curved Roof Structures", *International Balkans Conference on Challenges of Civil Engineering, BCCCE, 23-25 May 2013, Epoka University, Tirana, Albania*
- iv James N. Richardson, Rajan Filomeno Coelho, Sigrid Adriaenssens, "Robust topology optimization of truss structures with random loading and material properties: A multiobjective perspective", *Computers and Structures* 154 (2015) 41–47.
- v. *Practical aspect of structural optimization, 2nd Edition Released 06/2015 Altair University.*
- vi. *Practical aspect of finite element simulation, 3rd Edition Released 05/2015 Altair University.*