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# Analysis of single span prestressed concrete beams using different methods

**Rajesh Bhargava[1],K.K.Pathak[2], Saleem Akhtar[3]**
[1]Department of Civil Engineering, S. V. Polytechnic College, Bhopal (MP) India
[2]Department of Civil and Environmental Engineering, NITTTR, Bhopal (M.P) India
[3]Department of Civil Engineering UIT (RGPV) Bhopal (MP) India
E-mail: kkpathak1@rediffmail.com

***Abstract- In this paper the displacement behavior of single span prestressed concrete beams have been studied numerically by different methods. Six prestress concrete beams with parabolic curve cable profile for different amount of prestressing force were analyzed by Macaulay's method, Matrix method (STAAD.PRO) and FEA method. The results of all these three methods are compared with each other.***

**Keywords** Cable-profile; Prestressed concrete; Deflection: Friction; Finite element

## 1.Introduction

Applications of prestressed concrete are many, especially in recent times it has been widely used in bridges, buildings, rail sleepers, nuclear vessels, water and other liquid retaining structures etc. Prestressed concrete is a particular form of reinforced concrete in which external prestressing force is applied on the concrete to reduce or eliminate the tensile stresses and thereby control or eliminate cracking. Prestressed concrete is a typical set up of cable and concrete which makes a prestressed concrete section considerably stiffer than reinforced concrete section. In prestressed concrete, cable layout plays an important role in reducing tension from the concrete. Due to curvature, cable exerts forces on the concrete to counterbalance the forces causing tension. In curved tendons, upward force is imposed on the concrete which may reduce or eliminate the downward deflection as well; which is almost always the governing factor in structural design.

Chen et al. (2005) investigated the effective width of a concrete slab in steel–concrete composite beams prestressed with external tendons and concluded that prestressed with external tendons, the effective width of a concrete flange of a composite beam appeared slightly greater than the effective width when the beam was not prestressed. Özcan et al.(2006) carried out experimental and finite element analyses of the steel fiber-reinforced concrete (SFRC) beams. The results obtained from the finite element and experimental analyses were compared and found in good match. Chung et al.(2006) worked on the deflection estimation of a full scale prestressed concrete girder using long-gauge fiber optic sensors. It was demonstrated that long-gauge fiber optic sensors could provide the same accuracy with conventional sensors. Frederick et al.(2000) carried out experimental study of CFRP-prestressed high-strength concrete bridge beams. Fiber-reinforced polymer (FRP) tendons and reinforcing bars (rebar) were developed for use with concrete. FRP products are non-corrosive and lightweight when compared to traditional steel members. Zhang et al (2007) attempted experimental and theoretical studies on composite steel concrete box beams with external tendons. Sung et al.(2008) established stress–strain and deflection relationships of RC beam bonded with FRPs under sustained load Fiber-reinforced polymer (FRP) systems that had a strong resistance against long-term deformation. Padmarajaiah et al. (2000) attempted finite element assessment of flexural strength of prestressed concrete beams with fiber reinforcement. Influence of fibers on the concrete failure surface and stress–strain response of high strength concrete and the nonlinear stress–strain curves of prestressing wire and deformed bar were considered. Padmarajaiah et al.(2001) carried out flexural strength predictions of steel fiber reinforced high-strength concrete in fully and partially prestressed beam specimens. These studies mainly attempted to determine the influence of trough-shaped steel fibers in altering the flexural strength at first crack and check the load–deflection and moment–curvature characteristics, ductility and energy absorption capacity of the beams. Cattaneo et al.(2011) investigated the flexural behavior of reinforced, prestressed and composite self-consolidating concrete beams. The flexural behavior at service stage and at ultimate limit state was experimentally studied by means of four-point bending tests on six beams. Eythor et al.(2011) tested prestressed concrete beams with BFRP (basalt fibred reinforced polymer) tendons. The main findings were that the stiffness and bearing capacity of the beam increased relative to un-prestressed beams.

In this paper the displacement behavior of single span prestressed concrete beams have been studied numerically by different methods. Six prestress concrete beams with parabolic curve cable profile for different amount of prestressing force were analyzed by Macaulay's method; Matrix method (STAAD PRO) and FEM for different amount of prestressing force. The results of all these three methods are computed and results are critically analyzed and compared with each other.

## 2.Cable Profiles

Cables are laid as a parabolic curve for analysis purpose. The modeling of the prestressing cable in the finite element analysis is difficult and time consuming as concrete cable interaction involves various phenomena. The cable exerts longitudinal and transverse forces on the concrete due to friction and curvature. Also cable stiffness should be considered in FEA. Most of the researchers have idealized the cable as of parabolic shape. The reason is, for parabolic profile, curvature becomes constant and cable force can be represented as an equivalent uniformly distributed load acting in the opposite direction to the working loads. Although, parabolic assumption simplifies analysis.

The properties of the materials that were used are given in Table 1. A detailed summary of the concrete beams that were tested is given in Table 2. Three points loading adopted on all six beams in order to evaluate the deflection at mid span.

The load increment is taken 2 kN and it is applied on mid span section. Simply supported conditions are assumed for analysis. The load displacement results from all three methods are recorded and compared. The net deflection is calculated using Macaulay method analytical formula given below.

$$\Delta net = \frac{5PeL^2}{48EI} - \frac{WL^3}{48EI} - \frac{5wL^4}{384EI}$$

P = Prestressing force in N
e = eccentricity of prestressing force in mm
L = span of the beam in mm
E = modulus of elasticity of concrete in n/mm2
I = moment of inertia of beam about x-x axis in mm4
W = point load at mid span on beam in N
w = dead load of beam in N/mm

## 3.METHODS OF ANALYSIS

Numerical investigations of the six PSC beams have been carried out using following methods.

(A) Macaulay's method
(B) Matrix method (STAAD.Pro)
(C) Finite element method
(D)

### (A) Macaulay's Method

The double integration method is a technique used in structural analysis to determine the deflection of Euler-Bernoulli beams. Use of Macaulay's technique is very convenient for cases of discontinuous and/or discrete loading. Typically partial uniformly distributed loads (u.d.l.) and uniformly varying loads (u.v.l.) over the span and a number of concentrated loads are conveniently handled using this technique. Using calculus to find expressions for the deflection of loaded beams, it is normally necessary to have a separate expression for the bending moment for each section of the beam between adjacent concentrated loads or reactions. Each section will produce its own equation with its own constants of integration. It will be appreciated that in all but the simplest cases the work involved will be laborious; the separate equations being linked together by equating slopes and deflections given by the expressions on either side of each "junction point". However, a method devised by Macaulay enables one continuous expression for bending moment to be obtained, and provided that certain rules are followed the constants of integration will be the same for all sections of the beam. It is advisable to deal with each different type of load separately.

### (B) Matrix Methods of Analysis

Broadly the methods of analysis are categorized in two ways

1. Force Methods: Methods in which forces are made unknowns i.e Method of consistent deformation and strain energy method. In both these methods solution of number of simultaneous equations is involved.

2. Displacement Methods in which displacements are made unknowns i.e slope deflection method, Moment distribution method and Kani's Method. In slope deflection method also, the solution of number of simultaneous equations is involved. In both of the above methods, for the solution of simultaneous equations matrix approach can be employed and such method is called Matrix method of analysis. For matrix method based analysis, in this study, STAAD.Pro software has been used.

STAAD.Pro V8i is the most popular structural engineering software product for 3D model generation, analysis and multi-material design.

### (C) Finite Element Method

For numerical analysis, a house finite element method based FORTRAN code PRESS2D (Akhtar et al 2008) has been used. Concrete is modeled as 9 node plane stress element and cable is modeled as 3 node bar element. There are 4 elements and 27 nodes in the FE model is shown in Fig 3. The cable profile is moulded B-spline. The cable stiffness has been ignored in this study. As per IRC, coefficient of friction and Wobble effect are taken as 0.17 and 2x10-6 respectively (Bapat et al 2010). The deflection obtained from numerical analysis are given in Table 3 to 8

## 4. Result And Discussion

Using above methodology analysis of PSC beam A1 is done on applying prestressing force of 25.60 kN, For different loads corresponding deflections obtained. The load deflection data using all three methods is given in Table 3 and plot is given in Fig 4. It can be observed that all the results are quite close to each other.

Testing of PSC beam B1 is done on applying prestressing force of 28.48 kN, for different loads and corresponding deflections obtained. The load deflection data using all three methods is given in Table 4 and plot is given in Fig 5. It can be observed that all the results are quite close to each other.

Testing of PSC beam C1 is done on applying prestressing force of 35 kN, for different loads and corresponding deflections obtained, The load deflection data using all three methods is given in Table 5 and plot is given in Fig 6. It can be observed that all the results are quite close to each other.

Testing of PSC beam A2 is done on applying prestressing force of 42.04 kN, for different loads and corresponding deflections obtained. The load deflection data using all three methods is given in Table 6 and plot is given in Fig 7. It can be observed that all the results are quite close to each other.

Testing of PSC beam B2 is done on applying prestressing force of 44.96 kN, for different loads and corresponding deflections obtained. The load deflection data using all three methods is given in Table 7 and plot is given in Fig 8. It can be observed that all the results are quite close to each other.

Testing of PSC beam C2 is done on applying prestressing force of 51.36 kN, for different loads and corresponding deflections obtained. The load deflection data using all three methods is given in Table 8 and plot is given in Fig 9. It can be observed that all the results are quite close to each other.

It can be observed that the results obtained from FEA method show higher deflection than other two methods. Macaulay method gives lowest deflection. The variation between STAAD.Pro and Macaulay method is quite less as compared to variation between STAAD.Pro and FEM or Macaulay and FEM. The Matrix method results lie between analytical and FEM results. Macaulay method gives most conservative results.

## 5.CONCLUSION

In this study a comparison has been made between

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three popular methods of analysis i.e. analytical, matrix and FEA method, for the analysis of single span prestressed concrete beams. In analytical and matrix approach cable is modeled as parabola and in the case of FEA method, cable is modeled as B-spline since B-spline represents the actual cable profile. From the results of the six beams, it is observed that Macaulay method gives most conservative results. The analytical and matrix method results are close because in both method cable is modeled as parabolic.

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All dimensions are in mm

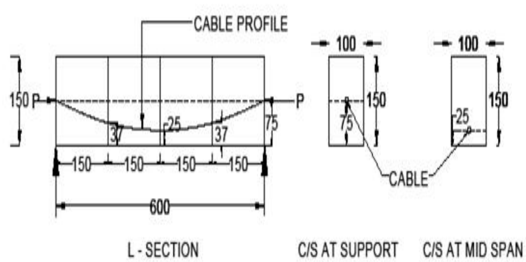


**Figure 1.** Cable profile and geometric details of Beam A1, B1, C1
All dimensions are in mm

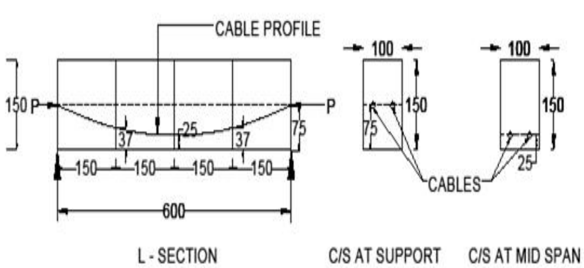


**Figure 2.** Cable profile and geometric details of Beam A2, B2, C2

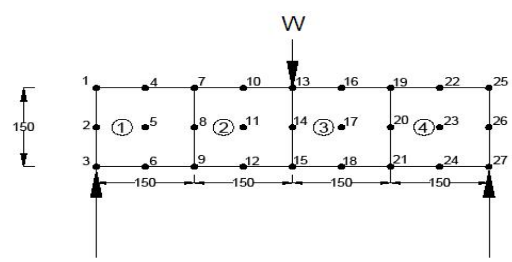


**Figure 3.** FE model for PSC single span beam
All dimensions are in mm

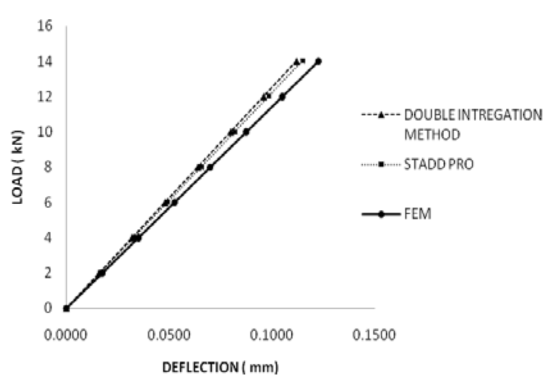


**Figure 4.** Load v/s Deflection for beam A1

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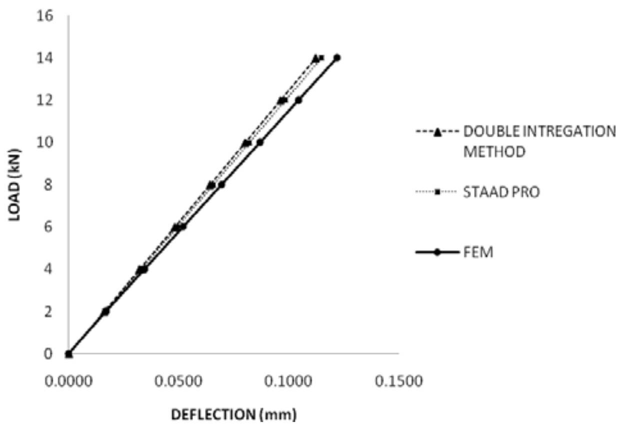


**Figure 5.** Load v/s Deflection for beam B1

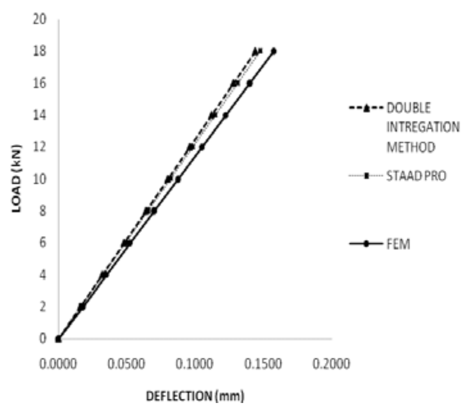


**Figure 6.** Load v/s Deflection for beam C1

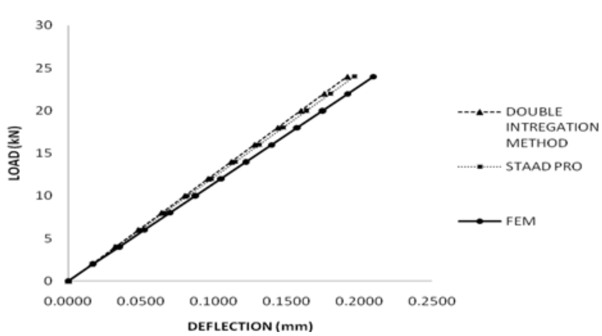


**Figure 7.** Load v/s Deflection for beam A2

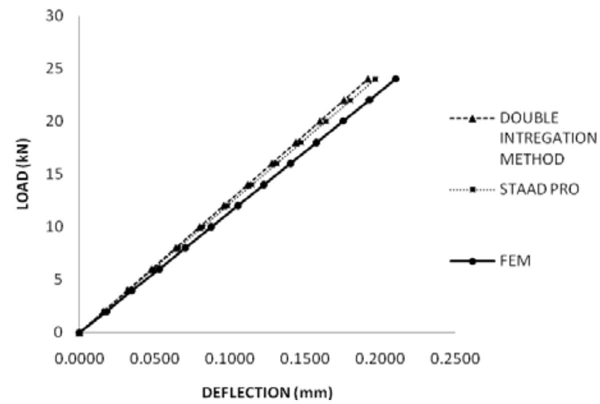


**Figure 8.** Load v/s Deflection for beam B2

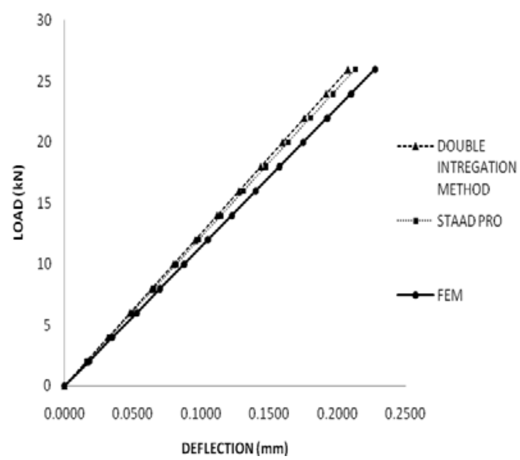


**Figure 9.** Load v/s Deflection for beam C2


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**Table 1.** Material properties of beam and prestressing cable

| Type | Cross sectional area (mm2) | Density (kN/m3) | Modulus of Elasticity(GPa) |
|---|---|---|---|
| Concrete Beam | 15000 | 25 | 20 |
| Strand | 21.2 | 7830.19 | 200 |

**Table 2.** Specimens prestressed with strand tendon

| Specimens | No. of strand | Prestressing Force(kN) | Elongation of Strand (mm) |
|---|---|---|---|
| A-1 | 1 | 25.60 | 4.57 |
| B-1 | 1 | 28.48 | 5.13 |
| C-1 | 1 | 35.00 | 5.40 |
| A-2 | 2 | 42.04 | 10.42 |
| B-2 | 2 | 44.96 | 11.18 |
| C-2 | 2 | 51.36 | 13.30 |

**Table 3.** Deflections under various loads for beam A1

| Load in kN | Numerical (FEM) Deflection in mm | Numerical (STAAD PRO) Deflection in mm | Numerical (Macaulay Method) Deflection in mm |
|---|---|---|---|
| 0 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0175 | 0.0164 | 0.0160 |
| 4 | 0.0350 | 0.0328 | 0.0320 |
| 6 | 0.0525 | 0.0492 | 0.0480 |
| 8 | 0.0699 | 0.0656 | 0.0640 |
| 10 | 0.0874 | 0.0820 | 0.0800 |
| 12 | 0.1049 | 0.0984 | 0.0960 |
| 14 | 0.1224 | 0.1148 | 0.1120 |

**Table 4.** Deflections under various loads for beam B1

| Load in kN | Numerical (FEM) Deflection in mm | Numerical (STAAD PRO) Deflection in mm | Numerical (Macaulay Method) Deflection in mm |
|---|---|---|---|
| 0 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0170 | 0.0164 | 0.0160 |
| 4 | 0.0345 | 0.0328 | 0.0320 |
| 6 | 0.0520 | 0.0492 | 0.0480 |
| 8 | 0.0695 | 0.0656 | 0.0640 |
| 10 | 0.0870 | 0.0820 | 0.0800 |
| 12 | 0.1044 | 0.0984 | 0.0960 |
| 14 | 0.1219 | 0.1148 | 0.1120 |

**Table 5.** Deflections under various loads for beam C1

| Load in kN | Numerical (FEM) Deflection in mm | Numerical (STAAD PRO) Deflection in mm | Numerical (Macaulay Method) Deflection in mm |
|---|---|---|---|
| 0 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0180 | 0.0164 | 0.0160 |
| 4 | 0.0351 | 0.0328 | 0.0320 |
| 6 | 0.0525 | 0.0492 | 0.0480 |
| 8 | 0.0700 | 0.0656 | 0.0640 |
| 10 | 0.0875 | 0.0820 | 0.0800 |
| 12 | 0.1050 | 0.0984 | 0.0960 |
| 14 | 0.1225 | 0.1148 | 0.1120 |
| 16 | 0.1399 | 0.1312 | 0.1280 |
| 18 | 0.1574 | 0.1476 | 0.1440 |

**Table 6.** Deflections under various loads for beam A2

| Load in kN | Numerical (FEM) Deflection in mm | Numerical (STAAD PRO) Deflection in mm | Numerical (Macaulay Method) Deflection in mm |
|---|---|---|---|
| 0 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0170 | 0.0164 | 0.0160 |
| 4 | 0.0350 | 0.0328 | 0.0320 |
| 6 | 0.0522 | 0.0492 | 0.0480 |
| 8 | 0.0697 | 0.0656 | 0.0640 |
| 10 | 0.0872 | 0.0820 | 0.0800 |
| 12 | 0.1047 | 0.0984 | 0.0960 |
| 14 | 0.1221 | 0.1148 | 0.1120 |
| 16 | 0.1396 | 0.1312 | 0.1280 |
| 18 | 0.1571 | 0.1476 | 0.1440 |
| 20 | 0.1746 | 0.1640 | 0.1600 |
| 22 | 0.1921 | 0.1804 | 0.1760 |
| 24 | 0.2095 | 0.1968 | 0.1920 |

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**Table 7.** Deflections under various loads for beam B2

| **Load in kN** | **Numerical (FEM) Deflection in mm** | **Numerical (STAAD PRO) Deflection in mm** | **Numerical (Macaulay Method) Deflection in mm** |
|---|---|---|---|
| 0 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0180 | 0.0164 | 0.0160 |
| 4 | 0.0350 | 0.0328 | 0.0320 |
| 6 | 0.0530 | 0.0492 | 0.0480 |
| 8 | 0.0702 | 0.0656 | 0.0640 |
| 10 | 0.0877 | 0.0820 | 0.0800 |
| 12 | 0.1051 | 0.0984 | 0.0960 |
| 14 | 0.1226 | 0.1148 | 0.1120 |
| 16 | 0.1401 | 0.1312 | 0.1280 |
| 18 | 0.1576 | 0.1476 | 0.1440 |
| 20 | 0.1751 | 0.1640 | 0.1600 |
| 22 | 0.1926 | 0.1804 | 0.1760 |
| 24 | 0.2100 | 0.1968 | 0.1920 |

**Table 8.** Deflections under various loads for beam C2

| **Load in kN** | **Numerical (FEM) Deflection in mm** | **Numerical (STAAD PRO) Deflection in mm** | **Numerical (Macaulay Method) Deflection in mm** |
|---|---|---|---|
| 0 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0180 | 0.0164 | 0.0160 |
| 4 | 0.0350 | 0.0328 | 0.0320 |
| 6 | 0.0530 | 0.0492 | 0.0480 |
| 8 | 0.0700 | 0.0656 | 0.0640 |
| 10 | 0.0876 | 0.0820 | 0.0800 |
| 12 | 0.1051 | 0.0984 | 0.0960 |
| 14 | 0.1226 | 0.1148 | 0.1120 |
| 16 | 0.1401 | 0.1312 | 0.1280 |
| 18 | 0.1576 | 0.1476 | 0.1440 |
| 20 | 0.1751 | 0.1640 | 0.1600 |
| 22 | 0.1925 | 0.1804 | 0.1760 |
| 24 | 0.2100 | 0.1968 | 0.1920 |
| 26 | 0.2275 | 0.2132 | 0.2080 |