Analysis of Composite Beams Using an Elasticity Method

Rakesh Patel[1], S.K.Dubey[2], K.K.Pathak[3]
[1]Department of Civil Engineering, SIRTS, Bhopal 462023, India
[2]Department of Civil Engineering, MANIT, Bhopal 462051, India
[3]Department of Civil & Environmental Engineering, NITTTR, Bhopal 462002, India.
Email- rakeshasct@gmail.com, dubeysk2001@yahoo.com,kkpathak1@rediffmail.com

Abstract- In this paper method of initial functions is used for the study of composite beam of two layers. Composite laminated beams are widely used in many structures, because this concept is very suitable for the development of lightweight structures. The equations of two dimensional elasticity have been used for deriving governing equations. No assumptions regarding physical behavior of beams are made. Most of the beam theories which are currently used for analysis involve assumptions regarding the physical behavior of beams. The beam theories which are based on assumptions are of a practical utility in the case of beams of moderate thickness.

Key words: Composite Beams, Method of Initial Functions, Elasticity Equations, stresses, displacements.

1. Introduction

Composite beams are beams that are built of more than one material. Examples are bimetallic beams, which consist of two different metals bonded together, sandwich beams and reinforced concrete beams. Composite beams may be analyzed by the same bending theory we used for ordinary beams, because the assumption that cross sections that are plane before bending remain plane after bending is valid in pure bending regardless of the material. In this present paper equations governing the flexure of composite laminated beams are derived without making any assumption regarding the physical behavior of beams. The method of initial functions (MIF) has been used for the purpose of deriving the equations.

Many researchers used method of initial function for the analysis of various structural members such as plates, shell, and beams etc. A mixed method is suggested for solving problems of theory of elasticity for thick plates as well as for the analysis of shells which is known as the method of initial functions [1]. It is used for two dimensional elasto dynamic problems for plain stress and plain strain conditions [2]. They have used MIF for the static analysis of simply supported, orthotropic, and laminated circular cylindrical shell of revolution subjected to axisymmetric load. By using the continuity conditions of displacements and stresses on each interface between adjacent layers, the state equation for the laminate is obtained [7].

MIF has been applied for deriving higher order theories for laminated composite thick rectangular plates [4]. The method of initial function is applied, to the flexural theory of circular plate subjected to ant symmetric lateral loads. The results are compared with solutions from classical theory [5]. They have used three-dimensional elasticity solutions for some static and dynamic problems of bending multi-layered anisotropic rectangular plates. They are derived by the method of initial functions [6].

Method of Initial Functions is used for the analysis of beams under symmetric central loading and uniform loading [3]. Governing equations are developed for composite laminated deep beams by using method of initial functions. The beam theory developed can be used for beam sections of large depth [8]. Applied this method for the analysis of orthotropic deep beams and compared the results with the available theory. It was observed that when depth increases MIF gives more accurate results [9]. Method of initial functions is used for the analysis of composite beams having three layers and developed governing equation [10].MIF is successfully applied for the analysis of reinforced concrete brick filled beams [12].An analytical approach for the analysis of vibration characteristics of honeycomb sandwich beams with multiple de bond at the interface between the carbon fiber reinforced plastic face sheets and honeycomb core is obtained and verified through test [11].

2. Formulation

In the case of a layered composite beam loaded at the top surface, the state of stresses and displacements at the free bottom surface of the beam is given by:

$$\{S_1\} = [u_1, \quad v_1, \quad 0, \quad 0]^T \tag{1}$$

Let

$$\{S_T\} = [u_T, \quad v_T, \quad Y_T, \quad X_T]^T \tag{2}$$

Where uT , vT ,YT and XT are the values of stresses and displacements at the top surface of the layered beam.

Relating the stresses and displacements at the top surface of the layer to those at the bottom surface by successive application of the transfer matrix [Li] across each layer, one obtains:

$$\{S_T\} = [A][S_1] \tag{3}$$

Where,

$$[A] = [L_N]_{y_N = h_N} \ldots\ldots\ldots\ldots\ldots\ldots [L_2]_{y_2 = h_2} . [L_1]_{y_1 = h_1} \tag{4}$$

The terms of the matrix [A] are evaluated after expanding the exponential in the form of a series.

The matrix has a form:

$$[A]=\begin{bmatrix} A_{uu} & A_{uv} & A_{uY} & A_{uX} \\ A_{vu} & A_{vv} & A_{vY} & A_{vX} \\ A_{Yu} & A_{Yv} & A_{YY} & A_{YX} \\ A_{Xu} & A_{Xv} & A_{XY} & A_{XX} \end{bmatrix} \quad (5)$$

The equation (3) relates the boundary conditions at the top surface to those at the bottom surface and is useful for deriving governing differential equations for a layered beam having a particular number of layers.

The method adopted for analyzing layered beams involves the determination of initial functions at the bottom surface of the beam by relating them through the matrix [A] to the stresses at the top surface.

3. Application To The Problem Of Composite Beam

A composite beam consists of the two layers. Therefore the matrix [A] becomes

$$[A]=[L_2]_{y_2=h_2}.[L_1]_{y_1=h_1} \quad (6)$$

Where h1 and h2 are the thickness of two layers.

The conditions at top are given by:

$$\{S_T\}=[u_T, \quad v_T, \quad -p, \quad 0]^T \quad (7)$$

Substituting the expressions (2) and (5) in the equations (3) we get:

$$A_{Xu}u_1 + A_{Xv}v_1 = 0 \quad (8)$$

$$A_{Yu}u_1 + A_{Yv}v_1 = -p \quad (9)$$

These equations are exactly satisfied by

$$u_1 = A_{Xv}\phi, \quad (10)$$

$$v_1 = A_{Xu}\phi \quad (11)$$

Where φ, is an unknown auxiliary function substituting the value of u1 and v1 from the equations (10) and (11) in the equation (9), the differential equation governing the problem of a normally loaded composite beam is obtained:

$$(A_{Yu}.A_{Xv} - A_{Yv}.A_{Xu})\phi = -p \quad (12)$$

The order of the governing differential equation (12) depends on the order of the terms in the matrix [A].

4. Solution of A Particular Problem

The following properties of isotropic material are taken:

E_1=10000 N/mm^2

E2=200000 N/mm^2

μ = 0.1, v = 0.3

The following beam dimensions are taken:
l = 3000mm, H = 310mm, h1= 300mm and h2= 10mm.
udl on the top surface of the beam is applied
p = 20 N/mm

5. Results and Discussion

The distribution of displacements and stresses across the thickness are presented in Fig. 1 to 5.

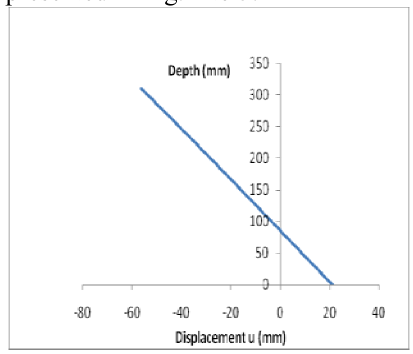

Figure 1. Variation of displacement (u) through the thickness of composite beam.

The value of displacement 'u' is more at the top surface and less at the bottom surface. Its variation is almost linear.

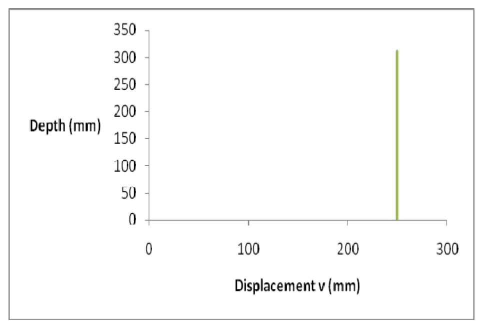

Figure 2. Variation of displacement (v) through the thickness of composite beam.

It can be seen from figure that displacement 'v' is uniform throughout the thickness.

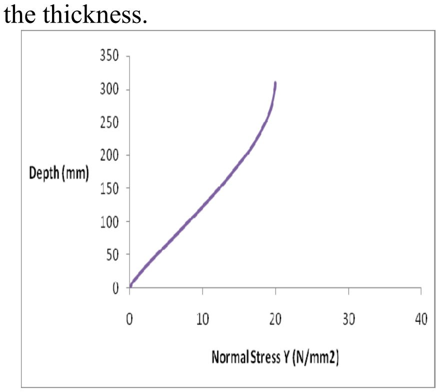

Figure 3. Variation of Normal stress (Y) through the thickness of composite beam

It is observed that the value of normal stress (Y) is zero at the bottom and maximum at the top of composite beam. The physical condition of normal stress equal to the applied normal load at the top is satisfied.

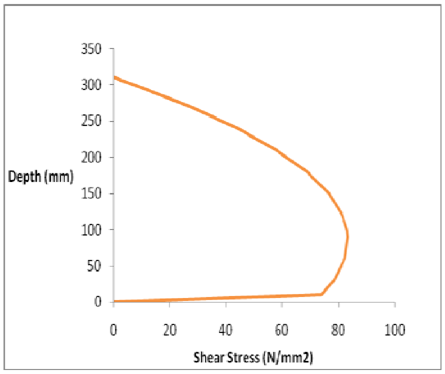

Figure 4. Variation of Shear stress (X) through the thickness of composite beam

It is seen from figure that shear stress (X) is more in the upper layer.

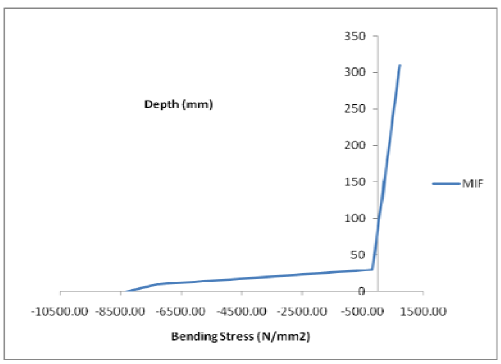

Figure 5. Variation of Bending stress (σx) through the thickness of composite beam

It is seen that the variation of bending stress across the depth is non linear.

6. Conclusion

The normal stress equals to the intensity of loading and shear stress equal to zero at the top of composite beam are obtain, this shows that MIF is successfully applied for the analysis of composite beam. The nature of the curves obtained for stresses and displacements is similar to those obtained by other theories. Hence it can be successfully used as an analytical approach for the analysis of composite beams.

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