International Journal of Engineering Research
Volume No.6, Issue No.5, pp :265-270

ISSN:2319-6890 (online),2347-5013(print)
1 May 2017

Markov Chain for Gibbs Sampler Applied in Electromagnetic Scattering by 2D Rough Surface

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Abstract-- The study of electromagnetic scattering by two-dimensional rough surface remains a challenging subjects for scientists due to its bidirectional scattering field computation. In this paper, Gibbs Sampler approach is applied to study electromagnetic scattering by two-dimensional rough surface. This work consists of generating a two-dimensional random rough surface. From that bivariate random distribution data generated, Gibbs Sampler method is applied to iteratively draw samples from the full conditional distribution. As we are dealing with a bivariate data two conditional probabilities distribution will be determined. They are respectively the probability distribution of x given y ($P(x|y)$) and the probability distribution y given x ($P(y|x)$). If sampling from those conditional distributions is possible then we can use Gibbs sampling method. For our study we consider a situation where it can be sampling from those two conditional distribution. The purpose of this sampling method is to estimate the marginal distributions $P(X)$ and $P(Y)$. And these marginal distributions are employed to compute the scattering field in x direction and in y direction. As results, a very high-frequency fluctuation is observed on scattering coefficient. Comparing with other works that exist literature this technique of Gibbs sampling shows an agreement.

Keywords: Gibbs Sampling, Electromagnetic scattering field, marginal and conditional probability distribution, 2D rough surface generation.

I. Introduction

The study of electromagnetic scattering has always been an interesting subject for researchers. For that reason, several techniques have been developed to calculate the scattering field. The classical analytic approaches of Kirchhoff approximation and Rayleigh-Rica small perturbation have been utilized to solve rough surface scattering issues [1], [2]], [13], [14]. But those two methods are limited in domain of validity. Nowadays with the high performance of the modern computer several techniques have been developed. These methods include Monte Carlo method and it has proven its effectiveness [15].
Numerical simulation of scattering by random media has also permitted to solve Maxwell's equations numerically without the limitations of analytical approximate models [3].

Gibbs sampling is a Markov Monte Carlo method for joint distribution estimation when the full conditional distributions of all concerned random variable are available. Gibbs sampling is a power technique employed to estimate complex models. It has many applications in medical data, biology of humans, and meteorology. Most recently this strategy is applied to analyze the gene expression data and the results have been able to be well interpreted [4], [5].

In this paper, we will apply Gibbs sampling to estimate two probabilities distribution function: marginal posterior distribution $P(X)$ and marginal posterior distribution $P(Y)$. The two distributions are utilized to analyze the scattering field phenomena by two-dimensional rough surface. Also to our knowledge this is a very first time that Gibbs sampling strategy is contributed to the analysis of electromagnetic scattering by two-dimensional rough surface.

II. 2D rough surface generation

Different techniques exist for generating two-dimensional random rough surface. A flexible and effective method utilized to generate random rough surface is the Discrete Fourier Transform. This technique is employed in this work to generate 2D rough surface. The surface parameters are given in Table1. More details are given on the mathematic expression developed to generate rough surface in
[6 - 8]. To generate the rough surface, MATLAB has helped for the implementation (see Figure 1). From this figure 1, $f(x,y)$ represents the surface height.

Table 1: 2D rough surface parameters values

Parameters	Number of surface points	rL-length of surface	h - rms height	clx and cly - correlation length (in x and y)
values	100	1λ	0.1λ	0.08λ

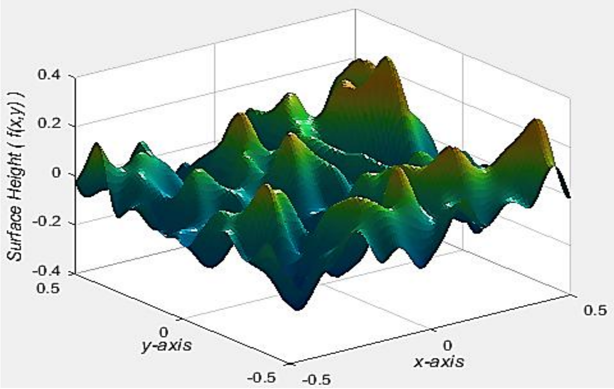

Figure 1: 2D Random Rough Surface.

III. Markov Chain for Gibbs Sampler

In this part of our work, we will introduce Gibbs sampling and its application to our bivariate rough surface data generated. Gibbs sampler is very popular Markov Monte Carlo (MCMC) sampling technique that allows one to avoid tedious computation. In this method each component is drawing independently of the others. Gibbs sampler is applicable only for certain types of problems, based on some criterions. For instant given a bivariate target distribution function which represents our two-dimensional rough surface characteristic [9], [10]:

- It is necessary to have the mathematic expressions of the conditional distribution of each variable in the bivariate joint distribution given other two variables in the joint.

-Also we have to be able to sample from each conditional distribution.

Given the target distribution of a bivariate normal distribution [11].

$$P(Z = f(x,y)|\mu, C) = \frac{1}{2\pi \det(C)^{\frac{1}{2}}} e^{\left(-\frac{1}{2}(Z-\mu)^T C^{-1}(Z-\mu)\right)} \quad (1)$$

where $Z = f(x,y), \mu = (\mu_x, \mu_y)$ and C represents 2 x 2 covariance matrix with diagonal entries σ_x^2 , σ_y^2 and off-diagonal $\sigma_{x,y}$. Also μ_x and μ_y are the mean values of x and y parameters.

$$C = \begin{bmatrix} \sigma_x^2 & \sigma_{x,y} \\ \sigma_{x,y} & \sigma_y^2 \end{bmatrix} \quad (2)$$

For Gibbs sampling we have the analytic expression of the two conditional distributions:

$$P(x|y^t) = N\left(\mu_x + \left(\frac{\sigma_{x,y}}{\sigma_y^2}\right)(y^t - \mu_y), \sigma_x^2 - \left(\frac{\sigma_{x,y}}{\sigma_y}\right)^2\right)$$

$$P(y|x^{t+1}) = N\left(\mu_y + \left(\frac{\sigma_{x,y}}{\sigma_x^2}\right)(x^{t+1} - \mu_x), \sigma_y^2 - \left(\frac{\sigma_{x,y}}{\sigma_x}\right)^2\right) \quad (3)$$

For the implementation of this technique, our target distribution is a bivariate normal distribution which represents the two-dimensional random rough surface distribution. From the surface data generated (target distribution) we draw samples.

The target distribution $P(Z = f(x,y)|\mu, C)$ is a Normal form with following parameterization:

$$P(Z = f(x,y)|\mu, C) = N(\mu, C)$$

with covariance

$$C = \begin{bmatrix} \sigma_x^2 & \sigma_{x,y} \\ \sigma_{x,y} & \sigma_y^2 \end{bmatrix} = \begin{bmatrix} 0.0109 & 0.0022 \\ 0.0022 & 0.0085 \end{bmatrix}$$

and mean

$$\mu = (\mu_x, \mu_y) = (-0.0720, 0.0868)$$

To get the two conditional distributions of variable x *and* y of the above expressions, we implement the Gibbs sampling. Therefore we have drawn 181 samples (See Figure 2).The implementation is performed on MATLAB and our goal here is to estimate the marginal distribution $P(X)$ and$P(Y)$. These are obtained by applying the following Gibbs sampling algorithm:

At time $(t+1)$ a new state for variable x conditioned on the most present state of variable y which is from the previous iteration (t). The same process is done by sampling a new state for the variable y conditioned on the most present state of the variable x , which is from current state $(t+1)$

Formally, the algorithm proceeds as follows:

1- First initialized x^0, y^0 to some values

2- Second

- Draw x^{t+1} from $P(x|y^t)$
- Draw y^{t+1} from $P(y|x^{t+1})$

where y^t is the previous state of the second dimension.

where x^{t+1} is the state of the first dimension after drawing from $P(x|y^t)$.

DOI : 10.5958/2319-6890.2017.00016.2

ISSN:2319-6890 (online),2347-5013(print)
1 May 2017

This completes one iteration of the Gibbs sampler, thereby producing one draw x^{t+1}, y^{t+1}.The above process is then repeated many times.

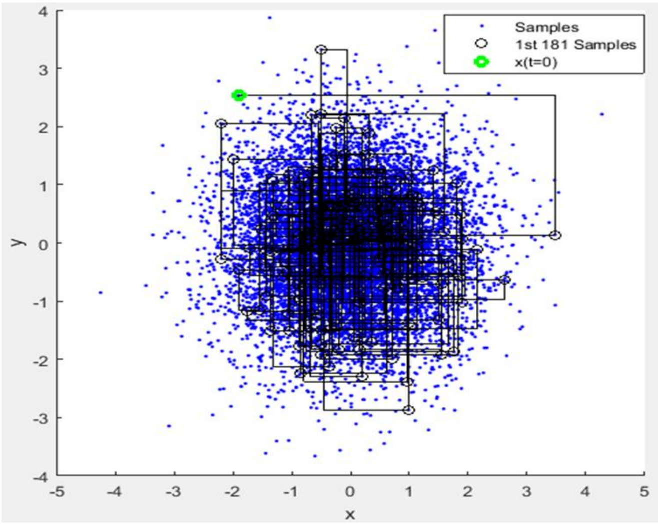

Figure 2: Gibbs sampling Markov and samples for 2D Rough surface Normal target distribution.

Figure 2 shows how the Gibbs sampling sequentially produces the values of $x \; and \; y$ from the target distribution $P(Z = f(x,y)|\mu, C)$. It is noticeable from this figure 2 that at each iteration Gibbs sampling technique first takes a step only along x direction then only along the y direction. After estimating $P(X)$ and $P(Y)$ we plotted them. And Figure 3 shows the comparison of the two distributions. Figure 4 and Figure 5 represent the individual plots of $P(X)$ and $P(Y)$. From those plots one can notice that their mean-values are slightly different.

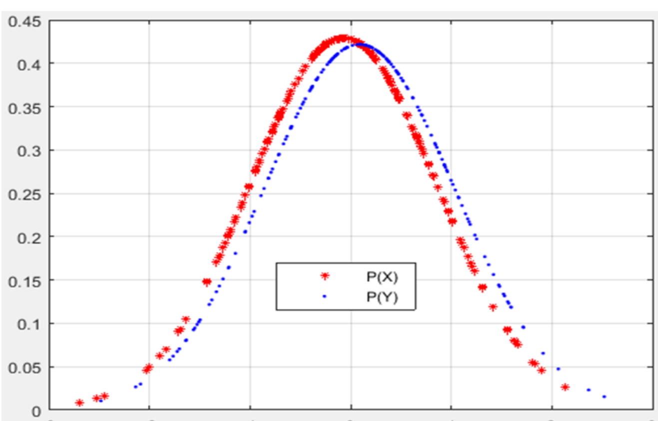

Figure 3: Comparison of Posteriors Marginal Probabilities Distribution ($P(X)$ and (Y)).

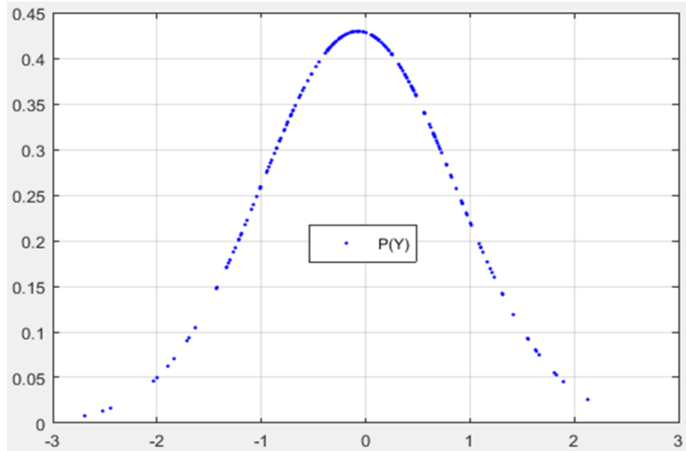

Figure 4: Marginal Probabilities Distribution ($P(Y)$).

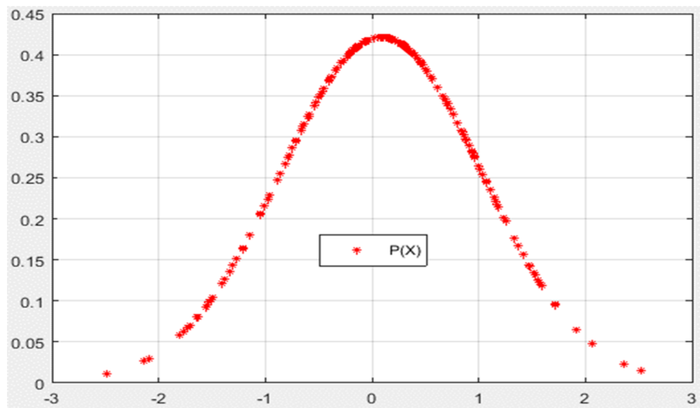

Figure 5: Marginal Probabilities Distribution (P(X)).

After estimating the marginal probabilities distribution, we can now deduct the joint probability, which represents our target bivariate distribution probability distribution (see Figure 6).

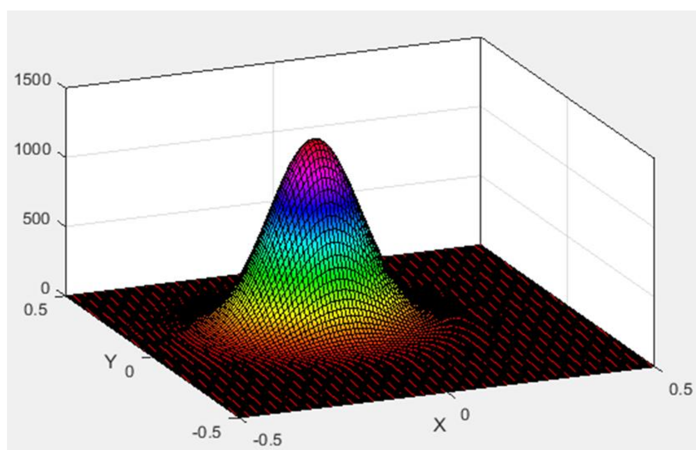

Figure 6: Target Joint Probability Distribution of $P(X)$ and P(Y).

IV. Computation and Plots of Scattering Fields

For this part of our work we will compute and plot the electromagnetic scattering field. In [12], the authors have demonstrated a method to calculate the scattering field of one-dimensional rough surface using the probability density function of the surface high in function of the scattering angle θ_s. For 2D case study they proposed to compute the probability distribution function (PDF) of the surface high in x and y directions in function of scattering angle θ_s. For our study $P(X)$ and $P(Y)$ are used to calculate the scattering field in x and y directions.

The scattering field equations is given by the following expression in [6 - 12].

$$E_s(\theta_s) = E_0 * F_s(\theta_i, \theta_s, n) * PDF * \int exp(-j\varphi(Z,\theta_i))\, dZ \quad (4)$$

where $F_s(\theta_i, \theta_s, n)$ represents Fresnel coefficient in function of incident angle θ_i, θ_s is scattering angle and n index of refraction. By applying Monte Carlo technique to transform an infinite integration to a finite integration will change the integration to summation.

$$\int exp(-j\varphi(Z,\theta_i)) * P(Z)\, dZ = \int \frac{exp(-j\varphi(Z,\theta_i)) * P(Z)\, dZ}{P(Z)} = \frac{1}{k}\sum_{1}^{k} e^{-j\varphi_k} \quad (5)$$

Hence equation (6) for scattering field in x direction becomes

$$E_x(\theta_s) = E_0 * F_s(\theta_i, \theta_s, n) * P(X) * \frac{1}{k}\sum_{1}^{k} e^{-j\varphi_k} \quad (6)$$

Similarly for the scattering field in y we have

$$E_y(\theta_s) = E_0 * F_s(\theta_i, \theta_s, n) * P(Y) * \frac{1}{k}\sum_{1}^{k} e^{-j\varphi_k} \quad (7)$$

The scattering coefficient is calculated by taking the ratio between the scattered power in a solid angle $d\theta$ around scattered angle θ_s and the incident power.

$$\sigma(\theta_s) = |E_s(\theta_s)|\,|E_s(\theta_s)|^* \quad (8)$$

V. Scattering Results and Comparisons

In this section we interpret the results of Gibbs sampler method for 2D rough surface electromagnetic scattering. Also we compare this method with others that already exist in the literature. From the Figure 6, it can be noticed that the scattering dynamic is slightly different and fluctuate a lot as a function of the scattering angle for both x and y scattering directions. The scattering coefficient obtained for scattering angle θ_s lower than 80 degree shows a maximum value at $-20\ dB$ and minimum value about $-80\ dB$. From scattering angle of 80 to 100 degree the scattering coefficient increases and reaches a maximum value about $-5\ dB$. For scattering angle θ_s greater than 100 we have a small decreasing of the scattering coefficient with same angular fluctuations.

The high-fluctuation obtained on the scattering coefficient is due to the sampling iteration from the rough surface data generated. The fact that the maximum value of the scattering coefficient remains almost unchangeable with respect to scattering angle (from 0 to 80 degree) is because of the convergence in Gibbs sampling technique. In this study as the surface roughness is very important, one can interpret that the high-frequency fluctuation observed on scattering coefficient is related to the surface roughness. Our method compared with one presents in [3] (see Figure 7) shows for both techniques the scattering field fluctuation is very important. Also the scattering fields varies mostly between $-60\ dB\ to -20\ dB$

In Gibbs sampling technique the maximum scattering coefficient ($-5\ dB$) is obtained at $\theta_s = 100\ degree$, whereas in Figure 7 the maximum is reached at $\theta_s = 40\ degree$ for both vertical and horizontal.

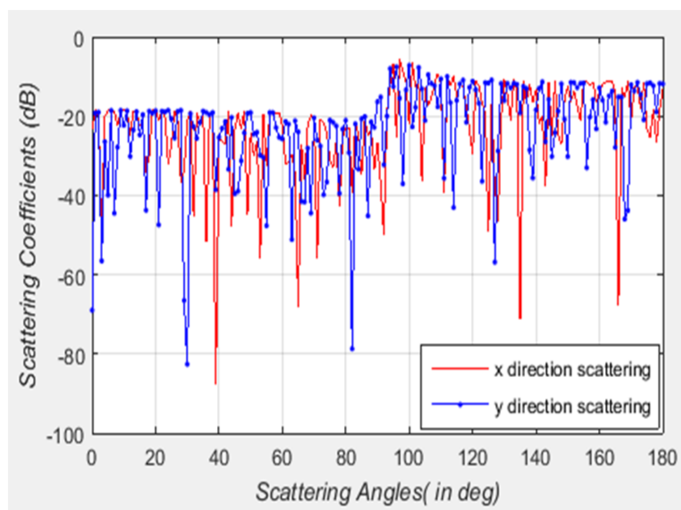

Figure 7: Comparison of Scattering Coefficient in x and y Direction.

ISSN:2319-6890 (online),2347-5013(print)
1 May 2017

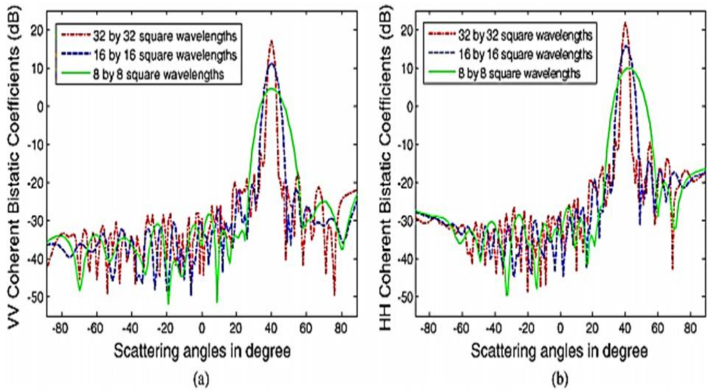

Figure 8: Comparison of coherent bistatic scattering coefficients in decibels for surface sizes of $8 \times 8\lambda^2$, $16 \times 16\lambda^2$, and $32 \times 32\lambda^2$. The relative permittivity is $8.35 + i1.99$, the rms height is 0.168λ, the correlation length is $10 \times \mathrm{rms}$ height, and 1000 realizations are averaged [3].

VI. Conclusion

This paper presents a new approach for the electromagnetic scattering analysis by 2D rough surface (Markov Monte Carlos of Gibbs sampler). We have employed the marginal probability distributions $P(X)$ and $P(Y)$ to compute respectively the scattering field in x and y directions. From the scattering coefficient plot one can notice a very high variation with respect to scattering angle. The variation of the scattering field lies in the fact that the roughness of the surface is very important. The scattering dynamic in x and y directions obtained are slightly different. Furthermore, in comparison with others methods developed in the literature, the Gibbs sampler shows an agreement. The results of this work demonstrate the capability of Gibbs sampling to model electromagnetic scattering from surfaces randomly rough in two directions (x and y).

Also the agreement between the results and the one in [3] (see Figure 7) shows the accuracy of Gibbs sampling approach of analysis of electromagnetic scattering phenomena by 2D rough surface.

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ISSN:2319-6890 (online),2347-5013(print)
1 May 2017

Monte Carlos Analysis of Electromagnetic Scattering by Random Rough Surface."

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