Abstract—Products are transported from depot to workstations and between workstations by milk-run vehicles in an automotive equipment manufacturer’s plant. Milk-run operators were completing routes earlier than anticipated time for each path in current state, and, also, transported quantities were non-standard. Manufacturer aims to optimization and standardization of milk-run tours. This paper presents suggestions to get over existing problems of a manufacturer. Finally, workload acquisition was obtained with the results of the suggestions.

Keywords – Gantt chart, mathematical model, milk-run optimization, mixed integer linear programming

I. INTRODUCTION

Fluctuations in national economies and in the global economy create significant risks for manufacturers and, nowadays, customers are demanding customized products in the right quantity at the right time. All of these features of today’s competitive environment have led manufacturers to make radical changes in their management and production structures. Manufacturers must also reduce costs to maintain their current market positions in this rapidly growing competitive environment.

This paper presents suggestions for an existing problem of an automotive equipment manufacturer in Turkey. The manufacturer is the leader in the market with 5000 employees. This study focused on the physical logistics area of the manufacturer facility. In physical logistics area, products are transporting from depots to the work stations or between workstations with milk-run vehicles. The milk run system consists of five different routes with different cycle times, as shown above:

- Green route (cycle time, 30 min),
- Yellow route (cycle time, 30 min),
- Purple route (cycle time, 60 min),
- Orange route (cycle time, 60 min),
- Blue route (cycle time, 60 min).

Four milk run operators are assigned per shift for those five routes. The existing problems of a manufacturer are:

- Milk run operators were completing tours earlier than determined lead-times.
- The workload of the milk-run operators was not equivalent.
- The variability of the transported quantities.
- Some work stations were served by more than one milk-run vehicle.

To get over existing problems two different suggestions are presented to the manufacturer. The first suggestion is to analyze routes by Gantt chart and the second suggestion is to optimize the routes by mixed integer linear programming model. The details of the suggestions and proposed models are presented in this study. The overall objective of the study is acquisition of workload.

II. LITERATURE REVIEW

In the competitive environment of the manufacturing, the elimination of activities that do not add value to the products gets much more important than it was before [1]. One of the most important wastes is the material handling waste within the seven wastes of the lean manufacturing [2]. If a good material handling system design is accomplished, it is claimed that the cost will decrease between 10% and 30% [3]. Effective and efficient management of transportation materials is becoming increasingly important both from the point of view of theoretical research and from the point of view of practical applications [4].

There can be a lot of applications, such as, design of the storages, material handling equipment selection and design of workflows, etc., for the elimination of waste in the lean logistics. Under the lean manufacturing conditions where pull system is applied and there is smooth manufacturing, a transportation vehicle called milk-run vehicle periodically moving on a predetermined route is regarded [1].

The milk-run times of the proposed system scheduled by Gantt charts. The Gantt chart is one of the most used planning and controlling tools in projects today and seen as a simple, intuitive, practical and useful tool to connect project activities and durations [5]. Gantt chart is a graphic representation of a project’s schedule, showing the sequence of tasks, which tasks can be performed simultaneously and the most critical tasks to monitor. The Gantt chart allows a user to avoid unrealistic timetables and schedule expectations, to help identify and shorten tasks that are bottlenecks, and to focus attention on most critical tasks.

The milk-run operator serves pre-defined customer nodes with a milk-run route and the tour starts from the depot and milk-run
vehicle visits a number of customer nodes located nearby in a given route [6]. Basically milk run cycle consists of the following steps [7]:

- Loading materials on means of milk-run vehicle,
- Transporting material to the point of use,
- Unloading material at the point of use,
- Loading empty handling equipment,
- Transporting and unloading empty handling equipment.

The vehicle routing problem (VRP) is the problem of minimizing the total travel distance of a number of vehicles, under various constraints, where every customer must be visited exactly once by a vehicle [8]. The VRP was first formulated by Dantzig and Ramser [9] and may be stated as follows: The problem includes a set of customers, each with a known location and a known requirement for some commodity, is to be supplied from a single depot by delivery vehicles of known capacity. Operational constraints (capacity and time) are considered while finding feasible routes. By considering additional requirements and various constraints on route construction, different VRPs have been formulated, such as, pick-up and delivery VRP, capacitated VRP, multiple depot VRP, VRP with time windows.

In general pick-up and delivery problem (GPDP), a set of routes has to be constructed to satisfy transportation requests. Transportation vehicle has a given capacity, a start location and an end location. Each transportation request specifies the size of the load to be transported, the locations where it is to be picked up and the locations where it is to be delivered [10]. Bianchessi and Righini [11] presented VRP with simultaneous pick-up and delivery as follows: the problem consists of V vehicles with identical capacity Q, set of N customers with integer pick-up and delivery demands and a matrix of distances between the locations where the customers and the depot are placed. The problem subject to following constraints:

- Each customer must be served only by one vehicle,
- Each vehicle leaves the depot carrying an amount of goods equal to the total amount it must deliver and returns to the depot carrying an amount of waste equal to the total amount it picked up,
- In each point along its tour each vehicle cannot carry a total load greater than its capacity.

III. SOLVING APPROACH FOR EXISTING PROBLEMS

To analyze the existing problems in current state value stream mapping (VSM) technique was used. VSM is a special type of flow chart that uses lean manufacturing symbols known to depict and improve the flow of inventory and information. The value added and non-value added processes are determined by VSM and it was seen that current lead times of the paths’ were not reflecting the reality. New lead times of the paths’ were computed with standard work study and workflows are revised.

Capacity utilization analysis is performed, to determine the amounts of materials transported with milk-run vehicles. In the current state, the lead time of the milk-run tours were recorded by the device. The lead time of the milk-run tour for each path for each day can be seen in Fig 1, Fig 2, Figure 3, Fig 4, and Fig 5. In Figs (1,2,3,4,5) vertical axis in left hand side represents the lead time of a milk-run tour and vertical axis in right hand side represents the amount of transported material per milk-run tour. The horizontal axis represents the tours in paths. The green horizontal line is the predefined lead time of a path, and horizontal red lines present the lower limit and upper limit of predefined lead time of a path.

As seen in Figs 1,2,3,4,5 the milk run operators were completing the tour earlier than predefined lead time. The lead times of the tours and transported quantities were not standard. Capacity utilization analysis tables are constructed to collect the data of transported quantities in each tour of related path. The columns of the tables represent the stops in milk run tour, and the rows of the table represent the tour info. By the help of the capacity analysis tables, picked–up and delivered quantities, empty tours, maximal and minimal transported quantities can be easily determined.
IV. DEVELOPED MODELS AND SUGGESTIONS

After determining data, two suggestions are presented to solve the existing problems of a firm. The first suggestion includes the analysing routes of five paths by Gantt charts and the second suggestion includes presenting a mixed integer linear programming model for route optimization.

**Suggestion 1: Route analysis**

The workflows of the five paths are revised with standard work study (chronometrage method). Capacity utilization analysis is performed to determine the workloads of the current state of five paths. The operating time of milk-run operator is 480 minutes per shift. In lunch and coffee break times, the tour is being taken by another milk-run operator; therefore, the milk-run tours are non-stop in all day. One operator for both blue path and orange path, one operator for purple path, one operator for green path and one operator for yellow path is assigned in current state, thus, four operators has been serving to five paths. Table 1 includes the capacity utilization ratio (CUR) of the current state after performing capacity utilization analysis.

The lead time, number of tours in one shift, the total time spent by milk-run operator and waste time in one shift are computed based on the data obtained from standard work study. The cycle time depicts the defined time interval of each tour and the lead time is the total spent time by milk-run operator for one tour.

### Table 1. Capacity utilization analysis of milk-run paths

<table>
<thead>
<tr>
<th>Cycle Time (min)</th>
<th>Lead Time (min)</th>
<th>No of tours in one shift</th>
<th>Total time for milk-run operator in tours (min)</th>
<th>Waste time in one shift (min)</th>
<th>CUR%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Current State</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blue</td>
<td>60</td>
<td>12</td>
<td>8</td>
<td>96</td>
<td>384</td>
</tr>
<tr>
<td>Purple</td>
<td>60</td>
<td>40</td>
<td>8</td>
<td>320</td>
<td>160</td>
</tr>
<tr>
<td>Yellow</td>
<td>30</td>
<td>22</td>
<td>16</td>
<td>352</td>
<td>128</td>
</tr>
<tr>
<td>Orange</td>
<td>60</td>
<td>30</td>
<td>8</td>
<td>240</td>
<td>240</td>
</tr>
<tr>
<td>Green</td>
<td>30</td>
<td>20</td>
<td>16</td>
<td>320</td>
<td>160</td>
</tr>
<tr>
<td><strong>Suggested System</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blue</td>
<td>60</td>
<td>12</td>
<td>8</td>
<td>96</td>
<td>384</td>
</tr>
<tr>
<td>Purple</td>
<td>60</td>
<td>40</td>
<td>8</td>
<td>320</td>
<td>160</td>
</tr>
<tr>
<td>Yellow</td>
<td>30</td>
<td>22</td>
<td>16</td>
<td>352</td>
<td>128</td>
</tr>
<tr>
<td>Orange</td>
<td>60</td>
<td>30</td>
<td>8</td>
<td>240</td>
<td>240</td>
</tr>
<tr>
<td>Green</td>
<td>60</td>
<td>26</td>
<td>8</td>
<td>208</td>
<td>272</td>
</tr>
</tbody>
</table>

Accoring to current state in Table 1:
- One operator is assigned for both blue and orange paths and assumed as a single path, so, computed CUR of the paths is 70% (20%+50%)
- One operator is assigned for yellow path and computed CUR is 74%.
- One operator is assigned for green path and computed CUR is 67%.
- One operator is assigned for purple path and computed CUR is 67%.

According to Gantt chart of the current system given in Fig 6, it was seen that, enhancement can be done for yellow and purple paths which have 67% CUR. However, purple path includes transportation between production stations and it is considered that, efficient outcomes cannot be taken for purple path. Therefore, improvement study was done on green path which includes transportation between depot and the work stations.

![Figure 6. Gantt chart of the current state](image)

The first suggestion is, to increase the cycle time of the green path to 60 minutes. Thus, the lead time of the green path is 26 minutes. According to Gantt chart of the suggested system given in Fig 7, one milk-run operator is assigned for both green and orange paths. The milk-run operator starts the tour for green path after completing the tour for orange path. One operator is assigned for blue and purple paths, namely, the milk-run operator starts the tour for blue path after completing the tour for purple path.

![Figure 7. Gantt chart of the suggested system](image)

The CUR of the current state and suggested state can be seen in Table 1, and according to suggested state in Table 1:
- One operator is assigned for both green and orange paths and assumed as a single path, thus, computed CUR of the paths is 94% (44%+50%).
- One operator is assigned for yellow path and computed CUR of the path is 74%.
- One operator is assigned for both purple and blue paths and assumed as a single path, thus, computed CUR of the paths is 87% (67%+20%).
**Suggestion 2: Route optimization model**

According to Pareto diagram, other criteria concerning to tour duration is, variability of the production quantities. The variability of the production quantities is caused by, different lead times of the tours and necessity of the stops of the milk-run vehicle in all workstations whether it has production or not. According to observations in workshop, different paths have similar routes, and some workstations have been served with different routes. Therefore, a mathematical model is presented for route optimization. Number of common workstations, route, capacity and cycle time are analysed for route optimization model. Parameters are analysed by factor analysis. Table 2 includes the relative weights of the parameters and Table 3 includes the eigenvector for each parameters.

### Table 2. Relative weights of the parameters

<table>
<thead>
<tr>
<th></th>
<th>Capacity</th>
<th>Cycle time</th>
<th>Route</th>
<th>No of common workstations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Cycle time</td>
<td>1/3</td>
<td>1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Route</td>
<td>1/5</td>
<td>1/4</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>No of common workstations</td>
<td>1/4</td>
<td>1/2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>TOTAL</td>
<td>1.78</td>
<td>4.75</td>
<td>12</td>
<td>7.5</td>
</tr>
</tbody>
</table>

### Table 3. Eigenvector value of the parameters

<table>
<thead>
<tr>
<th></th>
<th>Capacity</th>
<th>Cycle time</th>
<th>Route</th>
<th>No of common workstations</th>
<th>Eigenvector value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>0.56</td>
<td>0.63</td>
<td>0.42</td>
<td>0.53</td>
<td>0.54</td>
</tr>
<tr>
<td>Cycle time</td>
<td>0.19</td>
<td>0.21</td>
<td>0.33</td>
<td>0.27</td>
<td>0.25</td>
</tr>
<tr>
<td>Route</td>
<td>0.11</td>
<td>0.05</td>
<td>0.08</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>No of common workstations</td>
<td>0.14</td>
<td>0.11</td>
<td>0.17</td>
<td>0.13</td>
<td>0.14</td>
</tr>
</tbody>
</table>

According to eigenvector value of the parameters in Table 3, the most important criteria is capacity. In current state, the milk run vehicle includes maximum four wagons. Feasible paths for milk-run vehicle with four wagons are determined. The second important criteria in Table 3 is cycle time. To prevent the disruption in manufacturing, it is intended to combine the paths with similar cycle time. To determine the number of common work stations, pairwise comparisons is done for the routes. After the proposed investigations, it has been seen that green and yellow routes meet the all criteria. Fig 8 and Fig 9 include the paths for green route and yellow route.

In this paper, a mixed integer linear programming model is considered to construct a combined path for green path and yellow path to meet the deterministic demand of all workstations. A milk-run vehicle departs from the depot node and return to the depot node after serving all the work stations, each work station is visited at once, vehicles serve as simultaneous pick-up and delivery, and milk run vehicle cannot carry load more than capacity. The overall objective is to minimize the total distance of the combined path. The milk-run vehicles have homogenous wagons, fixed capacity, thus, serving green path with three wagons and serving yellow path with one wagon.

### Indices

- \( J \): Set of work stations (\( i,j = 1,2,\ldots,J \))
- \( J_0 \): Set of work stations with depot (\( i,j = 1,2,\ldots,J \))
- \( V \): Set of carried material type (\( v = 1,2,\ldots,V \))
- \( K \): Set of interjacent work stations (\( k = 1,2,\ldots,K \))
- \( v \): Material type indice

### Parameters

- \( C \): Capacity of wagon (m³)
- \( H_{A_{ij}} \): The volume of material type \( v \) picked up by work station \( j \)
- \( H_{B_{ij}} \): The volume of material type \( v \) delivered to work station \( j \)
- \( A_{ij} \): The quantity of material type \( v \) picked up by work station \( j \)
- \( B_{ij} \): The quantity of material type \( v \) delivered to work station \( j \)
- \( \text{MMM}_{ik} \): Precedence matrix
- \( d_{ij} \): Total distance between work station \( i \) and work station \( j \)
- \( M \): Very big number
- \( S \): Very big number
- \( n \): Number of work stations
- \( I \): The volume of vehicle load after the depot
- \( L_i \): The volume of vehicle load after the work station \( j \)
- \( u_{ij} \): Variable to eliminate the sub tour
- \( \tau_{ij} \): Time between work station \( i \) and work station \( j \)
- \( \tau_a \): Handling time of unit material at workstation
Decision variables

\[ x_{ij} = \begin{cases} 
1 & \text{if milk - run vehicle travels from workstation} \ i \ \text{to workstation} \ j \ (i \neq j) \\
0 & \text{otherwise}
\end{cases} \]

Model

\[ \min z = \sum_{i \in I_0} \sum_{j \in I_0} d_{ij} x_{ij} \]

Subject to

\[ \sum_{i \in I_j} x_{ij} = 1 \quad j = 1, 2, \ldots J \quad (1) \]

\[ \sum_{j \in I_i} x_{ij} = 1 \quad i = 1, 2, \ldots J \quad (2) \]

\[ \sum_{j \in I} x_{0j} = 1 \quad (3) \]

\[ \sum_{i \in I} x_{ij} = 1 \quad (4) \]

\[ \sum_{i \in I} x_{ij} = 1 \quad (5) \]

\[ \sum_{i \in I_j} x_{ijk} = \sum_{j \in I_k} x_{ijk} \quad k = 1, 2, \ldots J \quad (6) \]

\[ I = \sum_{i \in I_j} \sum_{j \in I} B_{ij} x_{ij} \quad (7) \]

\[ L_I \geq I + \sum_{j \in J} HA_{ij} - \sum_{j \in J} HB_{ij} - M(1-x_{ij}) \quad j = 1, 2, \ldots J \quad (8) \]

\[ L_I \geq I_v + \sum_{j \in J} HA_{ij} - \sum_{j \in J} HB_{ij} - M(1-x_{ij}) \quad i, j = 0, 2, \ldots J \quad (9) \]

\[ I \leq C \quad (10) \]

\[ L_I \leq C \quad (11) \]

\[ \sum_{j \in J} x_{ij} + \sum_{j \in J} x_{ji} + \sum_{d \in D} x_{ij} + \sum_{d \in D} x_{ji} + \sum_{d \in D} x_{ij} y \leq S \quad (12) \]

\[ \sum_{j \in J} j \cdot x_{ij} \leq \sum_{j \in J} j \cdot x_{ij} \quad (M M M \neq 0) \quad i, k = 0, 1, 2, \ldots J \quad (13) \]

\[ u_i \geq u_i + 1 - n(I-x_{ij}) \quad i, j = 1, 2, \ldots J \quad (14) \]

\[ u_i \geq 0 \quad i, j = 1, 2, \ldots J \quad (15) \]

\[ x_{ij} \in \{0, 1\} \quad i, j = 1, 2, \ldots J \quad (16) \]

The objective function (1) seeks to minimize total distance. Constraints (2) and (3) ensure that each work station is visited exactly once. Constraints (4) and (5) ensure the depart from and return to the depot. Constraints (6) ensure the depart from the visited work station. Constraints (7) ensure the vehicle load after the depot. The load balance of the vehicle is ensured by constraints (8) after first stop, constraints (9) for interjacent stops. The vehicle capacity is ensured by constraints (10) after first stop, constraints (11) for interjacent stops. Constraints (12) limit the time length of the milk run router by the cycle time. Constraints (13) ensure the precedence relations of the work stations. Constraints (14) eliminate the sub tours. Constraints (15) ensure \( u_i \) are nonnegative and constraints (16) define the binary variables \( x_{ij} \).

Proposed model is coded in MPL (Mathematical Programming Language) and solved optimally with GUROBI 5.1.0 solver. Fig 10 includes the new path of combined route of yellow route and green route based on the obtained optimal results.

**V. CONCLUSIONS AND DISCUSSION**

This paper aims to solve the real life problem of an automotive equipment manufacturer. The road-map of the proposed study is shown in Fig 11.

The total distance for yellow path and green path is 949.6 meters and 445.5 + 504.1 meters in current state, the total distance for combined path in suggested state is 673.45 meters. The acquisition from one tour is 376.15 meters. The combined path repeats 48 times per day, so, the total distance acquisition is 13255.2 meters per day.

It was observed that, the milk run tours were completing earlier than specified time in current state. In this study, the work flows of the routes are revised and the tours are scheduled with Gantt charts.
charts. According to new schedule, the cycle time of the green
 tour is increased to 60 minutes, and one operator is assigned for
 both orange path and green path. In current state milk run
 operator was starting the tour for blue path after completing
 orange path. In suggested state, one operator is assigned for both
 purple path and blue path and one milk run operator is assigned
 for yellow path.

To decrease the variability of the transported quantities, capacity
analysis tables are presented. Capacity analysis tables include
data of transported quantities for each day based on work stations
for each tours, thus, the deviations can be more easily detected.
Finally, mixed integer linear programming model is presented for
route optimization and solved optimally with MPL-Gurobi
solver. A new path is constructed by combining yellow path and
green path.

The contribution of this paper is multiple. Initially, the current
routes are scheduled by Gantt charts. Secondly mixed integer
linear programming model is presented for route optimization.
Gantt chart is an effective tool and is easy to use for real life
problems. The manufacturing firm can easily apply the method.
Mixed integer linear programming model solved route
optimization problem optimally by MPL. The manufacturing
firm exactly gets the optimal solution by mixed integer linear
programming model for route optimization model.

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