

Comparative Study of Design of Sewer Line Using Hazen-Williams and Manning Equations

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Abstract: *The design of gravity sewer and sewer line can be done using Hazen-Williams and Manning equations. The selection of pipe diameter depends upon sewer pipe materials, and minimum size, minimum and maximum velocities and slope; and for economical design, all these factors need to be considered. This paper deals with the optimal design of sewer line using Hazen-Williams and Manning equations as hydraulic model, and dynamic programming as optimization tool. The feasible set of diameter can be obtained considering the relative depth ratio, maximum and minimum velocities. The head loss is calculated for each diameter and the same is rounded off to the next higher value at an increment of 5 mm. The optimal solutions obtained using both the equations are presented in this paper.*

Keywords: *Dynamic programming, Hazen-Williams equation, Manning equation, Optimization, and Sewer design*

I. Introduction

Sewers collect waste water and foul sewage, transport sewage to a sewage treatment plant or other places of disposal. Wastewater collection systems contribute substantially to the overall cost of the municipal sewerage system, which requires a huge amount of investment for their construction as well as maintenance. A cost-effective design of the collection system will provide significant savings towards the cost of wastewater services. Pipes, manholes and excavations constitute a major portion of the cost of sewer line. Any economical design of sewer requires a selection of optimal depth-diameter combinations for all the links of a complete gravity wastewater collection system which cannot be achieved by engineering judgments. There are various optimization techniques in use to obtain the least-cost solutions among that dynamic programming is very common. Hazen-Williams (HWE) and Manning equations (ME) are generally used to design sewer line. Feasible set of diameter is calculated considering relative depth ratio, and maximum and minimum velocities. For each of the selected diameters, head loss in the sewer is calculated. Cost of a sewer includes cost of sewer pipe, cost of manholes and cost of excavation of sewer trench. The minimum solution cost is the optimal solution. Results of the design obtained using both the equations are compared and presented in this paper.

II. Review of Literature

The theoretical aspects regarding the design of sewers such as hydraulic equations for design of sewer, design constraints and relevant literature on the design of sewer line is given below.

2.1 Hydraulic Equations

The commonly used hydraulic equations for the design of sewers are as follows:

1. Manning equation;
2. Darcy-Weisbach equation; and
3. Hazen-Williams equation.

1. Manning Equation

The Manning equation is adopted for design of sewers flowing full as well as part because of its simplicity. However, the Manning formula with constant coefficient of roughness is applicable for a limited bandwidth, 0.004–0.04 of relative roughness (Christensen, 1984).

The Manning equation is given by:

$$V = \frac{1}{N} R^{2/3} S^{1/2} \quad (1)$$

Where,

- V = Velocity of flow, m/s;
 N = Manning's roughness coefficient;
 R = Hydraulic mean radius, m; and
 S = Slope of pipe.

The Manning equation is not a good representation of flow behavior in partly filled pipes, and this is true for the other conventional equations as well. Camp (1946) proposed correction factors to be applied to velocities calculated from the Manning equation. These factors vary with relative depth of flow. The curve shows that the velocity in the pipe flowing half-full is about 80% of that in the pipe flowing full, even though the hydraulic radius is the same for both conditions (Camp, 1946). Manning equation fails to cover the effect of the change in pipe diameter and flow characteristics on the roughness coefficient (Gupta, 1983). For the purpose of design of sewer the values of roughness coefficient N was taken as 0.015 in this presentation (Manual, 1993).

2.1.2 Darcy-Weisbach Equation

It is widely accepted that the Darcy-Weisbach equation for calculating head loss is a highly accurate pipe flow resistance equation (Liou, 1998). The Darcy-Weisbach equation is rational, dimensionally homogeneous, and applicable to other fluids as well as to water (Liou, 1998).

The Darcy-Weisbach equation is given by (Manual, 1993):

$$\text{---} \quad (2)$$

Where,

- h_f = Head loss, m;
 f = Friction factor;
 L = Length of pipe, m;

D = Diameter of pipe, m; and
 g = Acceleration due to gravity, m/s².

2.1.3 Hazen-Williams Equation

The Hazen-Williams equation is the better representation of flow behavior in pipelines under both conditions. Hazen-Williams formula can be expressed as (Manual, 1993):

(3)

Where, C_H = Pipe roughness coefficient.

The Manning equation is applicable for rough turbulent flow and the Hazen-Williams equation for smooth turbulent flow whereas the Darcy-Weisbach equation is applicable for laminar as well as turbulent flow (Liou, 1998).

2.2 Self-cleaning Velocity

A self-cleaning velocity may be defined as the velocity at which the solid particles will remain in suspension without settling at the bottom of the sewer. Also, it is the velocity at which even the scour of the deposited particles of a given size will take place. The self-cleaning velocity for the design of a sewer line is given in Table 1 (Swamee *et al.*, 1987).

Table 1: Adopted Self-Cleaning Velocity

Sewer diameter (m)	Self-cleaning velocity (m/s)
0.15 - 0.25	1.00
0.30 - 0.60	0.75
> 0.60	0.60

2.3 Scouring Velocity

At higher velocity, the flow becomes turbulent, resulting in continuous abrasion of the interior surface by the suspended particles. Hence, maximum velocity of flow is limited. The maximum velocity at which no scouring action or abrasion takes place is known as non-scouring velocity. Such velocity depends upon the material used for the construction of sewers. The scouring velocity for concrete pipe was taken as 3.0 m/s (Swamee *et al.*, 1987).

2.4 Relative Depth

To maintain free surface flow, sewers are designed to flow partially full. The maximum relative depth should not exceed 0.75 m (Manual, 1993). The relative depth varies with the diameter of pipes as given in Table 2 (Swamee, 2001).

Table 2: Relative depth

Sewer diameter (m)	Relative depth (η)
0.15-0.25	0.50
0.30-0.50	0.60
0.55-1.20	0.70
>1.20	0.75

2.5 Design of Sewer Line

Swamee *et al.* (1987) presented direct and simple method for sewer geometry for circular and non circular shapes for partly full flowing conditions. Methodology has been developed for satisfying the minimum and maximum velocity constraints.

Deshler (1986) developed a program titled sanitary sewer design (SSD) for the Apple II microcomputer. SSD is a heuristic program that attempts to find the least costly design of a sanitary

sewer network or pipeline. The program computes velocities, water depths, pipe slopes, and inverts corresponding to inputted diameters, flows, pipe lengths, ground elevations, and design criteria. SSD was used to find the least costly design of a 3.25 mile sewer trunk. It was also used to perform a sensitivity analysis, which evaluated the effect that design criteria, uncertainty in flows, and alteration of pipe diameters have on the cost of a sewer system.

Charalambous and Elimam (1990) developed a mathematical model containing a nonlinear convex function relating pipeline diameter and slope, which is approximated by piecewise linear segments. This approach uses a modified Hazen-Williams hydraulic model at part-full flow conditions, along with a newly developed universal expression to determine the coefficient of roughness. Moreover, the hydraulic formulation contains a regression equation to determine the Darcy friction factor based on the depth of flow in the pipe. The developed model has been extensively and successfully used to design several large sewer networks.

III. Hydraulic Equations

The sewers are designed running partially full. The central angle subtended at the centre is θ . The velocity ratio is defined as the ratio of design velocity of a sewer at partially full flowing condition to that at the full flowing condition of sewer. Using Hazen-Williams equation, velocity ratio which is denoted by k_v , is given by:

$$\frac{V_{partial}}{V_{full}} = \left(\frac{D_{partial}}{D_{full}} \right)^{4.75} \left(\frac{S_{partial}}{S_{full}} \right)^{0.0475} \quad (4)$$

The discharge ratio is defined as the ratio of design flow of sewer at partially full flowing condition to that at the full flowing condition of sewer. Using Hazen-Williams equation, discharge ratio which is denoted by k_q is given by:

$$\frac{Q_{partial}}{Q_{full}} = \left(\frac{D_{partial}}{D_{full}} \right)^{4.75} \left(\frac{S_{partial}}{S_{full}} \right)^{0.0475} \quad (5)$$

The velocity ratio k_v , using Manning equation is given by:

$$\frac{V_{partial}}{V_{full}} = \left(\frac{D_{partial}}{D_{full}} \right)^{4.75} \left(\frac{S_{partial}}{S_{full}} \right)^{0.0475} \quad (6)$$

The discharge ratio k_q , using Manning equation is given by:

$$\frac{Q_{partial}}{Q_{full}} = \left(\frac{D_{partial}}{D_{full}} \right)^{4.75} \left(\frac{S_{partial}}{S_{full}} \right)^{0.0475} \quad (7)$$

IV. Cost Considerations

The major components of sewers are sewer pipes, excavation of sewer trenches, and manholes which share a substantial portion of the overall cost of a sewer line. The cost structure for the following components of a sewer line is described as follows:

1. Sewer pipes;
2. Excavation of sewer trenches; and
3. Manholes.

4.1 Cost of sewer pipes

The cost of a sewer pipe for a given material varies with its diameter. The capitalized cost of a sewer C_m can be expressed as (Swamee, 2001):

(8)

Where, k_m, m = Cost parameters of pipe.

In order to determine the cost of pipe, the cost of pipes per metre length for RCC pipe was taken from Schedule of Rates (MJP, 2013). From the graph of diameter versus cost of pipes, the values of cost parameters for RCC pipe were found to be $k_m = 8561$ Rs./m, and $m = 1.478$ with $R^2 = 0.966$.

4.2 Cost of Excavation

The cost of excavation of a sewer trench consists of the cost of the following components:

1. Cost of earthwork; and
2. Cost of sheeting and shoring

The capital cost of the earthwork for a sewer, C_{ew} can be written as (Swamee, 2001):

$$(9)$$

Where, c_e = unit excavation cost at ground level, Rs./m³; c_r = increase in unit excavation cost per unit depth, Rs./m³/m; w = width of excavation trench, m; d_u = upstream depth of sewer, m; and d_d = downstream depth of sewer, m. According to Manual (1993), width w follows the following criteria:

$$w = 1, \quad D \leq 0.6 \text{ m} \quad (10)$$

$$w = D + 0.4, \quad D > 0.6 \text{ m} \quad (11)$$

In order to determine the cost of excavation, the cost of excavation per metre depth was taken from Schedule of Rates (MJP, 2013). From the graph of rate of excavation versus average depth of excavation, the value of cost parameters were found as $c_e = 124.6$ and $c_r = 14.06$.

The cost of sheeting and shoring of sewer trench depends upon the surface area of sidewalls of excavation trenches. The capital cost of sheeting and shoring of a sewer trenches, C_{es} can be written as:

$$(12)$$

Where, c_s = unit capital cost of sheeting and shoring at ground level, Rs./m²; and c_{rs} = increase in unit cost of sheeting and shoring per unit depth, Rs./m²/m. The value of cost parameters were found as $c_s = 185.5$, and $c_{rs} = 19.08$.

4.3 Cost of Manhole

The cost of manhole is an insensitive function of diameter, and it solely depends on the depth of the manhole. The capital cost of the manhole, C_h can be expressed as:

$$(13)$$

Where, d_h is depth of manhole. Value of cost parameter for manhole were found as $k_h = 13627$, and $b_h = -7247$ with $R^2 = 0.986$.

4.4 Design Procedure

The optimal design of a sewer line can be accomplished in 4 major steps as follows:

1. Determination of the set of feasible diameters;
2. Determination of head loss;
3. Determination of upstream and downstream depths of sewers; and
4. Determination of the total cost of sewer.

V. Illustrative Design Example

Dynamic programming technique has been adopted for the optimal design of a sewer line. The process of optimal design of

a sewer line is illustrated using a 2-link sewer line design example. The data adopted for L_i , Q_i and Z_i have been used to design the 2-link sewer line, and is given in Table 3. The description of a sewer is shown in Fig. 1. In this case, the minimum velocity was adopted from Table 1. The Hazen-Williams coefficient of roughness for RCC pipe was taken as 140 (Manual, 1993).

Table 3: Design Data for 2-Link Sewer line

Link	Q (m ³ /s)	Z ₁ (m)	Z ₂ (m)	Length(m)
1-1	0.045	160.00	159.60	75
1-2	0.200	159.60	159.30	80

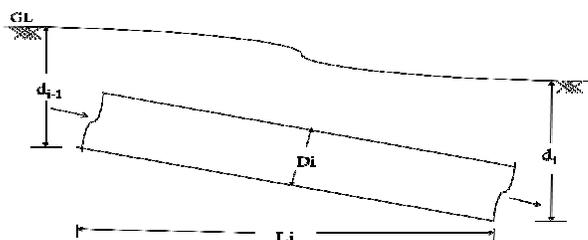


Figure 1: Schematic diagram of a sewer

The 2-link sewer line problem was solved by both Hazen-Williams and Manning equations. The set of feasible diameters along with the corresponding head loss obtained from Hazen-Williams and Manning equations results are given in Table 4.

Table 4: Set of feasible diameter with head loss

Pipe No.	q (m ³ /s)	Diameter (m)	Head loss (m)	
			HWE	ME
1	0.045	0.350	0.100	0.215
		0.300	0.205	0.480
		0.250	0.850	2.290
		0.200	2.500	7.520
2	0.200	0.750	0.025	0.050
		0.700	0.040	0.070
		0.650	0.055	0.105
		0.600	0.080	0.165
		0.550	0.120	0.260
		0.500	0.285	0.665
		0.450	0.470	1.165
		0.400	0.850	2.180

Figures 2 and 3 shows the variation of head loss versus diameter for the flows 0.045 and 0.200 m³/s, respectively.

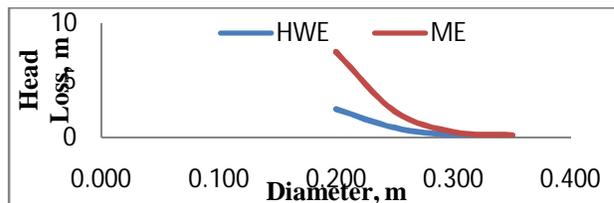


Fig. 2: Variation of head loss versus diameter ($Q = 0.046$)

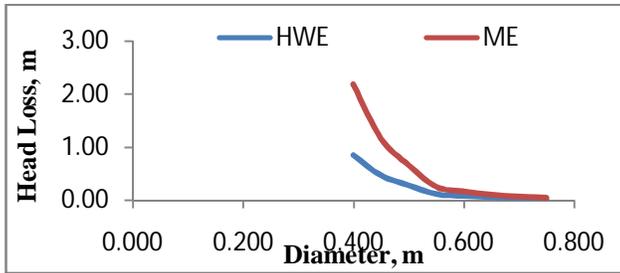


Fig. 3: Variation of head loss versus diameter ($Q = 0.2$)

From Figures 2 and 3, it can be observed that the head loss obtained using Manning equation is more as compared to that with Hazen-Williams equation. The total cost of sewer line was calculated for each individual combination of selected diameter of sewers. The optimal solution of sewer line is given in Table 5.

Table 5: Optimal solution of sewer line

Option No.	Optimal diameter (m)	Depth (m)		Head loss (m)	Cost (Rs. Lakhs)
		d_u	d_d		
Hazen-Williams equation					
16	0.300	1.495	1.300	0.205	4.58
	0.400	1.400	1.950	0.850	
Manning equation					
15	0.300	1.300	1.380	0.480	5.08
	0.450	1.530	2.395	1.165	

The optimal cost obtained from Hazen-Williams equation is 4.58 Rs. Lakhs. The variation in the cost of sewer line versus option number is shown in Fig. 4. The optimal cost obtained from Manning equation is 5.08 Rs. Lakhs. The variation in the cost of sewer line versus option number is shown in Fig. 5.

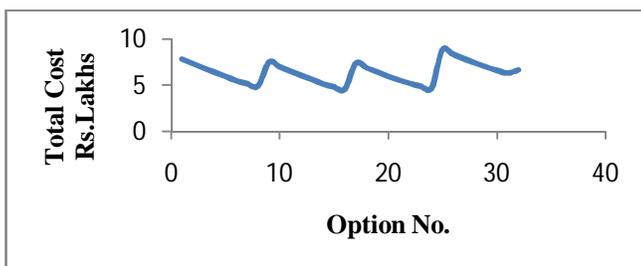


Fig. 4: Total cost versus option number (HWE)

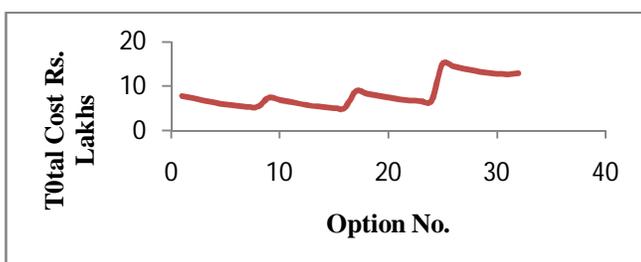


Fig. 5: Total cost versus option number (ME)

The cost of components for the optimal solution obtained in Table 6. The optimal cost obtained using Manning equation is more than that obtained using Hazen-Williams equation

Table 6: Cost parameter of sewer line

Description	Cost (Rs. Lakhs)	
	HWE	ME
Pipe	2.85	3.19
Excavation	1.28	1.39
Manhole	0.44	0.50
Total	4.57	5.08

VI. Conclusions

Optimal design of a sewer involves determination of combination of slope and diameter so as to obtain the least cost design along with the satisfaction of various constraints. The non-linear cost function subjected to a set of non-linear constraints makes the sewer design problem more complex to handle analytically. From the study of optimal design of sewer line using dynamic programming carried out in this paper, the following conclusions can be drawn:

1. The optimal solution can be easily obtained using dynamic programming.
2. The overall saving in the design of a sewer can be obtained as compared to the conventional design.
3. The optimal design obtained with Manning equation is 11.2% more costly as compared to that with Hazen-Williams equation.

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